MATHEMATICS ACTIVITIES

Grade 4 or Grade 5

TEACHER'S GUIDE

LEVEL 9

Mandated Course of Study

OFFICE OF
CURRICULUM and INSTRUCTIONAL DEVELOPMENT
THE SCHOOL DISTRICT OF PHILADELPHIA

MATHEMATICS ACTIVITIES

Teacher's Guide

LEVEL NINE

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CONTENTS

FOREWORD

TO THE TEACHER OF MATHEMATICS IN YEARS 1-8:

This is one of 20 activities booklets, one for each level, that have been prepared to help teachers to present to children the mathematics of Levels I through XX for grades 1-8.

The mathematics curriculum for grades 1-8 is divided into 20 teaching levels. This organization is designed to improve instruction and to foster the continuous development of children. Each level embodies carefully delineated areas of learning arranged in progressive stages. Such an arrangement of sequential skills and subject matter eliminates grade restrictions and permits continuous growth according to the individual's ability and rate of learning.

The activities in these booklets are designed to make mathematics interesting, motivating, and enjoyable to students. The activities reflect the instructional philosophy of moving a student from the concrete experience through the abstract application of mathematics concepts and skills and provide ample opportunity to do so.

Tests are also available to teachers to help them discover at which level each child can work; pupil progress can be determined and evaluated. Together, the mathematics activities booklets and the evaluation tests provide important assistance in inducing mathematical learning in the children through this levels approach. No single overall extensive, complete guide is available to teachers to support this important learning process--nor is it considered desirable to produce such a publication at this time. It has been determined that an effective way to attain the stated objectives is to develop for teachers, and hence for children, activities booklets to supplement and expand the mathematics concepts as stated in the scope and sequence.

There is evidence that children can learn mathematics through a discovery approach with the use of physical materials in a laboratory setting. The activities included in these booklets are consistent with the learning process techniques found to be effective today.

The activities included in these booklets have been compiled by classroom teachers, consultants, the mathematics collaborating teachers, and the mathematics supervisors. Clearly, these activities should not be considered to be complete or closed. Teachers are encouraged to consult textbooks, teacher manuals and other resources for additional activities than can be included in the teaching process. Also, to introduce each of seven areas in each booklet, the scope of that area is included to assist teachers to anticipate the topics that will be considered.

This revised mathematic curriculum is the result of the ongoing evaluation carried out by the Division of Mathematics Education and is responsive to changes in mathematics method and content nationwide. It was coordinated by the Elementary Mathematics Curriculum Committee under the sponsorship of the Office of Curriculum and Instruction.

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NUMERATION

1.20 <u>Decimal System of Numeration</u> a. Review renaming numbers. b. Review the characteristics of the decimal system.	

1.20 Decimal System of Numeration

a. Review renaming numbers.

- Provide many opportunities for pupils to learn that a number has many names.
- Select a number and have students think of many ways to express it, for example, the number 256:

  200 + 50 + 6

  two hundred fifty-six

  2 hundreds, 5 tens, 6 ones

  1 hundred, 15 tens, 6 ones

  2 hundreds, 4 tens, 16 ones

  2 (100) + 5 (10) + 6

  300 - 44

  125 + 131

  512 ÷ 2

- Express numbers in expanded notation form: e.g., 1462 = 1000 + 400 + 60 + 2

674	1208	8075
509	2456	5007

- Express the following as standard numerals:

  2 hundreds, 14 tens, 7 ones = 347

  5 thousands, 26 hundreds, 1 ten, 8 ones =

  6 thousands, 9 hundreds, 12 tens, 2 ones =

- Rename 25, using the set of whole numbers and the addition, subtraction, multiplication, and division operations. A typical solution for this problem is:

Addition	Subtraction	Multiplication	Division
20 + 5 = 25	40 - 15 = 25	5 x 5 = 25	75 ÷ 3 = 25
3+5+8+9 = 25	50 - 25 = 25	25 x 1 = 25	25 ÷ 1 = 25
5+5+5+5+5 = 25	300 - 275 = 25	5 x (4 + 1) = 25	100 ÷ 4 = 25
10 + 15 = 25	525 - 500 = 25	(3 + 2) x 5 = 25	200 ÷ 8 = 25

b. Review the characteristics of the decimal system.

- Discuss with pupils the principle that our system of numeration uses ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These ten symbols, or digits, are used to write any numeral in our system.

- Display a place-value chart to the class. Discuss the value of the numeral 7 as it moves down the chart:

Hundreds	Tens	Ones
		7
	7	◌
7	◌	◌

Ask pupils to explain <u>why</u> the 7 increases in value. Points which should be elicited at this time are:

1. Grouping is by tens; the base of the system is ten.
2. Each place or position has a value ten times greater than the place to its immediate right.
3. Zero is used primarily as a place holder. It is also a cardinal number, representing the concept of "not any".

- A concrete review of grouping and place-value concepts can be presented with a brief chip-trading-up activity using a 10:1 exchange and three columns. (Chip-trading activities have been presented in earlier levels.)

- Using a deck of digit cards, ask a pupil to draw two at a time (e.g., 5 and 3) and tell the class the largest and smallest numbers possible. Do this several times. Increase the number of cards drawn to three, and ask pupils to continue to create the largest and smallest numbers possible. Record the numbers on a place-value chart either on the board or the overhead screen.

Note: These numbers can be used for computation practice + or −.

WHOLE NUMBERS

2.40-2.50 Addition and Subtraction

a. Estimate answers before computations.

2.40 Addition

a. Continue to develop addition of 3-digit numbers (without and with regrouping).

b. Explore the properties of addition.

(1) closure
(2) commutative
(3) associative
(4) identity

2.50 Subtraction

a. Extend concepts.

(1) inverse of addition
(2) finding missing addend

b. Continue to develop subtraction of 3-digit numbers (with regrouping)

c. Test subtraction for properties.

2.60-2.70 Multiplication and Division

a. Estimate answers before computations.

2.60 Multiplication

a. Develop multiplication of 3-digit numbers by 1-digit numbers (without and with regrouping).

b. Explore properties of multiplication.

(1) closure
(2) commutative
(3) associative
(4) identity
(5) distributive

2.70 Division

a. Review concepts

(1) inverse of multiplication
(2) repeated subtraction
(3) missing factor
(4) measurement and partition concepts

b. Develop division of 3-digit numbers by 1-digit numbers

(1) by estimation
(2) zeros in the quotient
(3) writing remainders
(4) checking

2.75 Mathematical Sentences

a. Develop the language and skills for solving mathematical sentences.

(1) symbols
(2) open sentence
(3) equalities and inequalities
(4) true and false sentences

2.40-2.50 Addition and Subtraction

a. Estimate answers before computations.

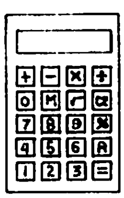
- Encourage pupils to estimate answers before actually computing. Rounding to the nearest tens or hundreds is an important first step in the estimation process.

Activity examples:

Problem:	Rounded:	Estimated Sum:
49 + 33	50 + 30	80

Problem:	Rounded:	Estimated Sum:
496 + 253	500 + 250	750

Problem	Estimate	Calculation	Reasonable Estimate?
427 + 660	1100	1087	Yes
408 - 89	300	319	Yes
59 + 74	200	133	No
94 - 58	30	36	Yes



The calculation to verify the estimate may be done on the calculator.

2.40 Addition

a. Continue to develop addition of 3-digit numbers (with and without regrouping).

- This concept was introduced in level VIII. Provide students with opportunity to review and practice both types of addition with 3-digit numbers.

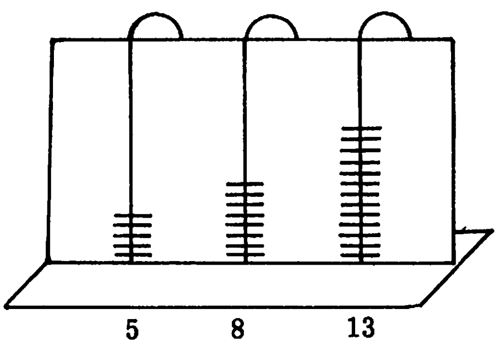
- Review place value; use instructional aids if needed.

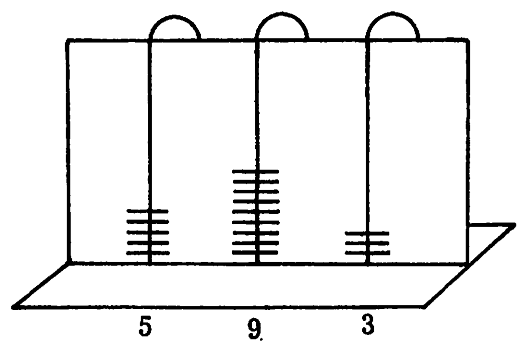
No regrouping	hundreds	tens	ones	
423	4	2	3	400 + 20 + 3
+ 164	1	6	4	100 + 60 + 4
587	5	8	7	500 + 80 + 7 or 587

- With regrouping, ones only:

  Use the appropriate form.

366	300 + 60 + 6
+ 227	200 + 20 + 7
593	500 + 80 + 13 or
	500 + 80 + 10 + 3 or
	500 + 90 + 3 = 593





- With regrouping in both ones and tens places, use forms or instructional aids described in previous section.

  Example:

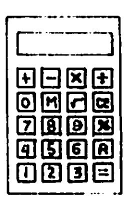
```
  439
+ 384
  823
```

  Work toward the short form; the preceding activities are introductory and serve to strengthen understanding of the process.

- Find the missing digits:

```
  5 □ 2
+ □ 8 □
  9 3 2
```

  Each placeholder represents a different number.



Use the calculator to have pupils check their calculations.

- Continue to use horizontal form.

  246 + 322 = ________

- Choose any 3-place number, for example 432. Make up all possible 2-digit numbers from 432:

  43
  34
  42
  24
  32
  23

  Add these numbers: 198

  Divide this sum by the sum of the original digits: 4 + 3 + 2 = 9

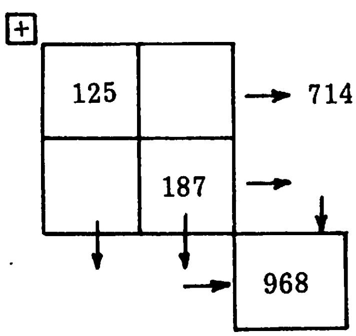
  198 ÷ 9 = 22

  Add digits of original number until a one-digit answer is obtained.

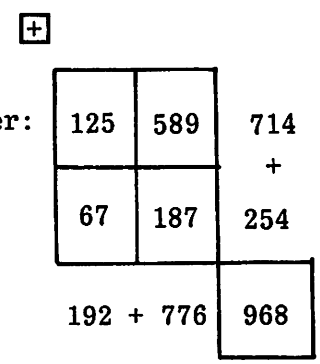
  Example: 987: 9 + 8 + 7 = 24; 2 + 4 = 6

  Answer to division is always 22.

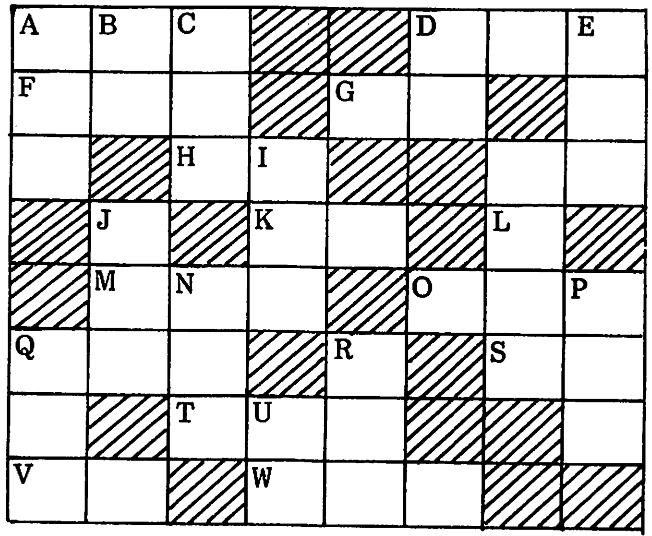
  Try using many different numbers.



Answer:



• Solve the cross-number puzzle:



Across

A	145 + 197
D	239 + 325
F	561 + 129
G	4 tens + 9 ones
H	296 + 447
K	43 + 46
M	250 + 387
O	365 + 386
Q	325 + 199
S	25 + 18
T	369 + 207
V	14 + 15
W	387 + 42

Down

A	56 + 306
B	36 + 13
C	147 + 60
D	34 + 25
E	239 + 161
G	265 + 174
I	396 + 91
J	512 + 450
L	329 + 325
M	28 + 34
N	96 + 249
P	76 + 58
Q	387 + 145
R	681 + 281

b. Explore the properties of addition.

These properties have been developed gradually in levels I - VIII. In level VIII, the commutative, associative, and identity properties are explored. In the development of each concept below, stress is to be placed on what happens to the numbers and why, not on memorization of terms. Do not have pupils memorize the names of the properties. Concepts will be tested by application.

(1) Closure

This property refers to the fact that when two whole numbers are added, the answer is a whole number.

Ask pupils to find two whole numbers which, when added together, do not result in a whole number. Besides having an opportunity for practice of addition, pupils will observe closure in that they will not be able to find two whole numbers which do not result in a sum of another whole number.

Examples of closure:

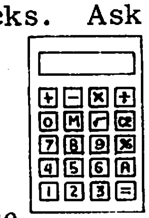
Addition pairs	Whole-number sum
7, 9	16
49, 21	70
0, 6	6
100, 95	195

(2) Commutative

The commutative property of addition states that the order of the addends will not affect the sum $(a + b = b + a)$.

Develop with pupils a list of practical, daily occurrences in which the order of operations does not affect the outcome. Examples: putting on shoes: left one first or right one first - it doesn't matter; eating breakfast: east toast first or drink juice first - it doesn't matter. Then, ask for activities where order does matter, such as putting on shoes and socks. Ask pupils to discuss the difference.

- Fill in the missing numbers.



Check by using the calculator.

362 + 729 = 729 +

___ + 513 = 513 + 169

249 + ___ = 423 + ___

```
  574
+ 609
1,183
```

Add down. Check by adding up.

This process illustrates commutativity.

(3) Associative

This property states that grouping of addends does not affect the sum; that is:

$(a + b) + c = a + (b + c)$. Demonstrate this property with small numbers:

$$3 + (2 + 7) = (3 + 2) + 7$$
$$3 + 9 = 5 + 7$$
$$12 = 12$$

and with large numbers:

$$(247 + 168) + 309 = 247 + (168 + 309)$$
$$415 + 309 = 247 + 477$$
$$724 = 724$$

Asking pupils to verify examples similar to those above provides worthwhile drill activities.

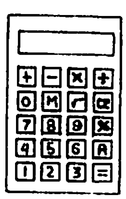
(4) Identity

This property states that using zero as one of two addends results in the sum being identical to the other addend ($a + 0 = a$; $o + a = a$).

- Discuss the word identity. Develop a list of real-life situations where the identity of a person or thing is unchanged after an action has occurred. Examples: A student puts on different clothes each day; he is still the same person. A house at 2030 Broad Street receives a fresh coat of paint; it is a different color, but it is still the same house at the same address.

- Have students perform computations on problems such as

$7 + 0 = \square$ $\quad 78 + 0 = \square$ $\quad 0 + 106 = \square$



Use the calculator with + 0 as the constant. Put in any number; push the [=] key.

Find the missing numbers:

$$\begin{array}{r} 3\ 6\ 9 \\ +\ 2\ \square\ \square \\ \hline 5\ 6\ 9 \end{array} \qquad \begin{array}{r} 2\ \square\ 6 \\ +\ 1\ 2\ \square \\ \hline 3\ 2\ 6 \end{array}$$

2.50 Subtraction

a. Extend concepts.

(1) Inverse of addition

Review and expand the idea that subtraction is the "taking apart" of a group and addition is the "putting together" of a group.

- Develop a list such as the following with your class:

Doing	Undoing
Filling a pail of water	Emptying a pail of water
Opening a window	Closing a window
Putting on shoes	Taking off shoes

- Application to algorithm:

Develop sets of DOING and UNDOING algorithms from groups of three numbers.

Examples: 25, 19, 6

Doing	Undoing
19 + 6 = 25	25 - 6 = 19
6 + 19 = 25	25 - 19 = 6

- Present these problems:

1. 11 + 12 = 23
2. 23 - 12 = 11

Ask: "What if the 23 in the second equation were replaced with 11 + 12 ?"

3. (11 + 12) - 12 = 11

Ask: "What if 11 in the first equation were replaced with 23 - 12 ?"

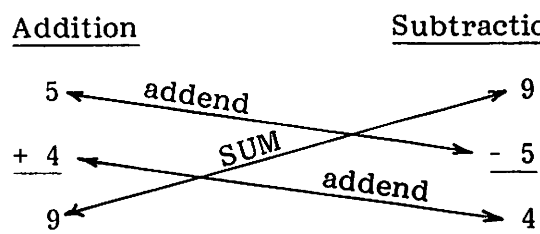
4. (23 - 12) + 12 = 23

Explain how the addition and subtraction of 12 in equations 3 and 4 do and undo each other.

Present more examples.

(2) Finding the missing addend

- Review the parts of addition and subtraction problems in vertical form. Show the correspondence of parts.



- Encourage pupils to solve problems of this type and discuss the correspondence of parts.

6	9	15	21	Note here that finding
+ 3	- 6	+ 6	- 15	a difference is the
□	□	□	□ ;	same as finding a
6	9	15	21	missing addend.
□	- 6	+ □	15	
9	□	21	□	

These problems can be written horizontally, for example:

6 + □ = 9   15 + □ = 21

- Lead the students to generalize that, if one of two addends and their sum are given, we must subtract the given addend from the sum to find the difference.

6 4 3	5 8	Place missing
- 2 □ 1	- □ 6	digit in various
3 7 □	3 □	positions.

At first, use no regrouping. Later, regrouping examples can be used.

- Relate the above to problems where two addends are given and the sum is missing.

a.	□	6	c.	□	14
	- 6	+ 3		- 14	+ 7
	3	□		7	□

b. □ - 6 = 3; 6 + 3 = □   d. □ - 14 = 7; 14 + 7 = □

b. Continue to develop subtraction of 3-digit numbers (with re-grouping).

This concept was developed in level VIII. Provide pupils with opportunity to review and practice subtraction of 3-digit numbers.

c. Test subtraction for properties.

1. Closure: Ask whether it is possible to find whole-number solutions to problems such as the following:

$$3 - 9 = \square$$

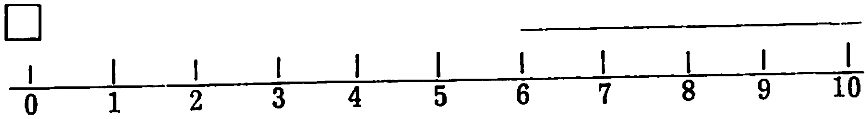
$$4 - 10 = \square$$

Pupils should be able to generalize that a solution from the set of whole numbers is NOT possible. Therefore subtraction is not CLOSED on the set of whole numbers.

2. Commutativity

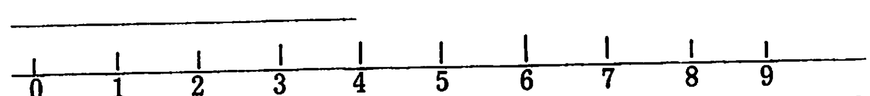
Use numberlines to show that subtraction is not commutative on the set of whole numbers.

$10 - 4 = \square$



10 - 4 = 6

$4 - 10 = \square$



Pupils should discover that this subtraction cannot be represented on a numberline which begins with zero.

3. Associativity

Have pupils solve the following mathematical sentences:

(a) $(9 - 4) - 2 = \square$

(b) $9 - (4 - 2) = \square$

Ask whether the solution for (a) is the same as that for (b). The answer is no: (a) = 3; (b) = 7.

Work on a few similar sets of problems which will lead pupils to generalize that:

Subtraction is not associative on the set of whole numbers.

4. Identity

Review the concept of an identity element. For addition, 0 is an identity because a + o = o + a for any whole number a.

When considering whether subtraction has an identity element, pupils are likely to say it does if they consider only 5 - 0 = 5. For an operation to have an identity element, it must work from both the right and the left. Consider 0 - 5 = □ . There is no whole number that we can put into the box to make the equation true. Therefore, subtraction has no identity element.

2.60-2.70 Multiplication and Division

a. Estimate answers before computations.

Encourage pupils to estimate answers before actually computing. Rounded numbers facilitate estimating, since they are easily computed mentally.

Examples:

1. 215 → 200, x 6 → x 6

2. 497 → 500, x 8 → x 8

2.60 Multiplication

a. Develop multiplication of 3-digit numbers by 1-digit numbers (without and with regrouping).

1. Review multiplication of 2-digit numbers by a one-digit number without and with regrouping (see Level VIII).

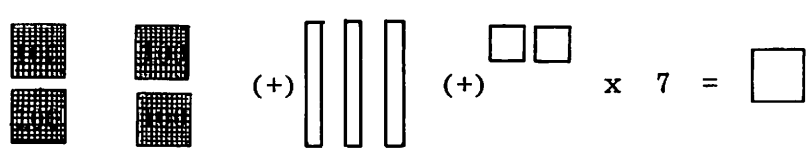
2. The teacher may wish to show pupils this multiplication explanation relating arrays and expanded notation to the multiplication algorithm. This is not to be tested. It is for explanation and development only.

Problem: 432 x 7 = □

Expanded form:

(400 + 30 + 2) x 7 = □

Pictorial:



Example:

```
    4 3 2
  x     7
  -------
      1 4 ──────→ 7 x 2
    2 1 0 ──────→ 7 x 3 0
  2 8 0 0 ──────→ 7 x 4 0 0
```

3. The conventional algorithm should be introduced with successive levels of complexity.

<u>Example</u>:

```
  1 3 2
x     2     (without regrouping)
-------

  1 3 6
x     2     (with one regrouping)
-------

  2 4 2
x     7     (with 2 regroupings)
-------

  3 0 6
x     4     (zeros in the hundreds place)
-------
```

4. Have the students find missing factors in examples such as these.

```
 □ 7 2      4 3 □      □ □ 4      2 4 7
 x   4      x   2        x 3      x   □
 -----      -----    -------      -----
 6 8 8      8 6 6    1 5 4 2      4 9 4

 4 □ □
 x   2
 -----
 9 2 2
```

5. Ask pupils to find the missing product in the following table:

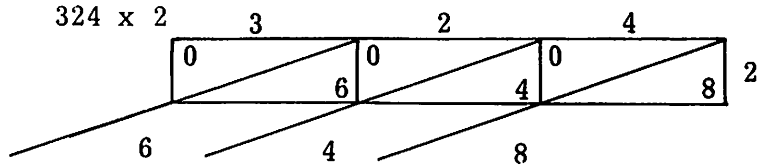
factor	1 8 4	7 1 4	6 2 3	4 0 8	5 1 5
factor	2	3	4	4	5
product					

6. Make two-inch-square digit cards with the numerals 1 to 9. Shuffle the cards and place them face down on the desk. Turn over three cards. Then turn over one more and multiply to find the product. After doing this once, reshuffle the cards and set up a new problem. Pupils may record each problem and find the digits which gave the largest and the smallest product.

[2] [7] [4]
x 5
―――――

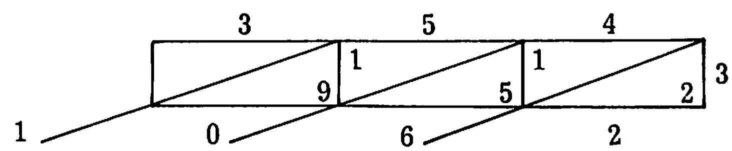
7. Show examples of lattice multiplication without regrouping:

324 x 2



The answer is 6 4 8.

and with regrouping: 3 5 4 x 3



The answer is 1 0 6 2.

b. Explore properties of multiplication.

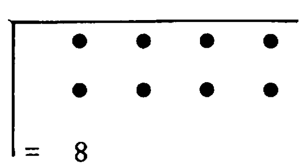
(1) closure - Lead pupils to discover that when any two whole numbers are multiplied, the product is a whole number.

7 x 3 = 2 1

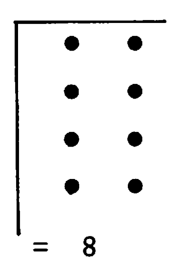
4 x 9 = 3 6

(2) commutative - Have pupils show by means of an array that the order of the two factors in multiplication does not affect the product.

2 x 4



and 4 x 2



Pupils will see that 2 x 4 = 4 x 2. After developing this approach with pupils, have them construct arrays by drawing dots or circles, or by pasting paper circles on construction paper to demonstrate the following equations:

5 x 9 = 9 x 5

8 x 7 = 7 x 8

15 x 3 = 3 x 15

1 x 7 = 7 x 1

As pupils engage in this activity, the following idea should be emphasized: in multiplication, changing the order of the factors will not affect the sum.

(3) <u>associative</u>: Place three factors on the board to be multiplied, e.g., 3 , 5 and 8 . These numbers may be grouped or associated as follows: (3 x 5) x 8 or 3 x (5 x 8). Use the parentheses to show how the factors are to be grouped with multiplying.

(3 x 5) x 8	3 x (5 x 8)
15 x 8	3 x 40
120	120

Through experiences of this type, the pupils should be led to generalize that the way we group the factors will not affect the product.

Give pupils other sets of numbers such as

4 , 2 , 7

2 , 5 , 6

7 , 3 , 2

Have the numbers grouped and multiplied. Check to make sure that the product does not change as long as the same three numbers are multiplied, no matter how they are grouped.

(4) <u>identity</u>: Recall that the identity for addition is zero. Help pupils discover that the identity for multiplication is 1. The teacher may use the multiplication table and mathematical sentences to help pupils become aware of the identity element. Look at the equation 7 x ☐ = 7. The only number that can be used to complete it is 1. Have the students look at the identity element in various positions.

16 x 1 = 16

1 x 47 = 47

$$\begin{array}{r} 58 \\ \times\ 1 \\ \hline 58 \end{array}$$

1 x 119 = 119

(5) distributive: This property is the mathematical basis for computational procedures involving multiplication of whole numbers and is also a justification for many computational shortcuts. Place this example on the board: 5 x (4 + 6). What is happening here is that 5 will be multiplied by 4 and then by 6. The two products are then added together. Multiplication distributes itself over addition.

5 x (4 + 6)

(5 x 4) + ((5 x 6)

20 + 30

50

- When multiplying larger numbers, pupils will find it helpful to use this property.

```
 3 6    =   3 0   +   6
x  7        x         7
           2 1 0 + 4 2 = 2 5 2
```

or

7 x 3 6 = □

= 7 x (3 0 + 6) ⟶ renaming

= (7 x 30) + (7 x 6) ⟶ distributive property of multiplication over addition

= 2 1 0 + 4 2 ⟶ multiplication

= 2 5 2 ⟶ addition

- Ask pupils to distribute one of the factors in the equations below and find the solution:

8 x 7 = 8 x (5 + [2]) then (8 x [5]) + (8 x [2])

9 x 14 = 9 x (+ □) then (9 x □) + (9 x □)

7 x 43 = 7 x (+ □) then (7 x □) + (7 x □)

- Have pupils supply the missing numerals in examples such as these:

	Solutions:
7 x 3 = (4 x 3) + (□ x 3)	□ = 3
4 x 5 = (□ x 5) + (1 x 5)	□ = 3
27 x 6 = (20 x 6) + (7 x □)	□ = 6
69 x 4 = (60 x 4) + (□ x 4)	□ = 9

2.70 Division

a. Review concepts

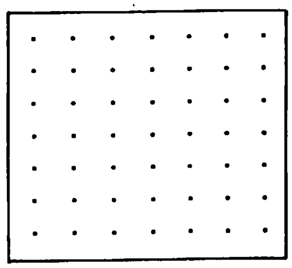
(1) inverse of multiplication

Review with pupils that multiplication and division are doing and undoing operations.

doing	undoing
6 x 7 = 42	42 ÷ 7 = 6

Pupils will realize that this relation means that, if they know the basic multiplication facts, they also know the basic division facts.

- Give pupils blank cards and ask them to draw a rectangular array of dots on the card.



Have them exchange cards and write the two multiplication and the two division equations about the array of dots:

doing	undoing
6 x 7 = 42	42 ÷ 7 = 6
7 x 6 = 42	42 ÷ 6 = 7

(2) repeated subtraction

- Review the idea of division as repeated subtraction. (See Level VIII.)
- Ask pupils to use subtraction to see how many 6s are in 48:

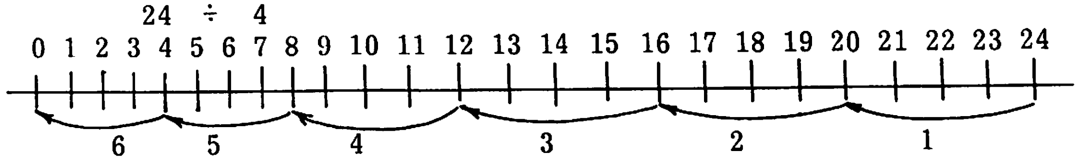
$$\begin{array}{r} 48 \\ -\ 6 \\ \hline 42 \\ -\ 6 \\ \hline 36 \\ -\ 6 \\ \hline 30 \\ -\ 6 \\ \hline 24 \\ -\ 6 \\ \hline 18 \\ -\ 6 \\ \hline 12 \\ -\ 6 \\ \hline 6 \\ -\ 6 \\ \hline 0 \end{array}$$

- Have pupils use repeated subtraction to solve the following:

  4/36   9/45   8/56

- Use the numberline also to show division as repeated subtraction.

  24 ÷ 4



- Use a calculator with a constant key for subtraction to solve each of these: 7/21   3/54   4/40

  The number of times the equal key is pressed until zero is reached would be the answer.

(3) missing factor

- Make flash cards with factors missing from multiplication sentences. Have students work in pairs, orally, to build up speed and accuracy.

  6 x □ = 42   □ x 9 = 63   8 x □ = 56   □ x 3 = 27

- Make charts such as the one below to find missing factors.

	4	9	6	6	6	4	5	5	9	5
x	3	3	4	3	6	9	7	6	4	4
	12	27	24	18	36	45	35	30	36	20

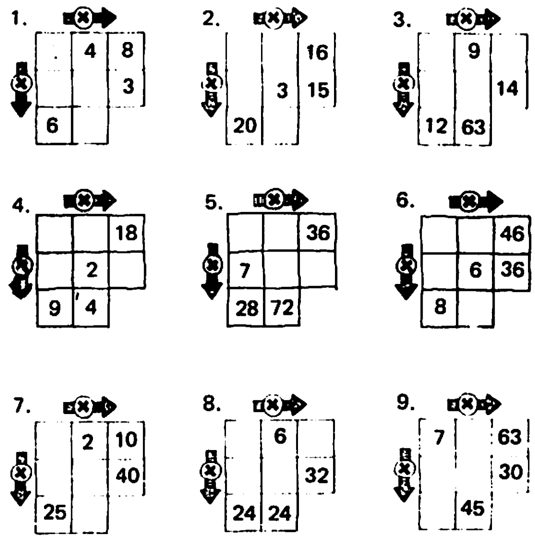
	2	7	4	8	5	8	9	9	6	9
x	7	3	7	4	8	3	3	2	9	4
	14	21	28	32	40	24	27	18	54	36

- Give squares such as the following for continued practice in finding missing factors:

4	2	8
1	3	3
4	6	24

3	6	18
	2	8
12	12	144

Other examples:



- Find missing factors by supplying the related multiplication algorithm:

27	-	□	=	9	because	9	x	□	=	27
54	-	6	=	□	"	□	x	6	=	54
36	-	□	=	4	"	4	x	□	=	36
18	-	□	=	2	"	2	x	□	=	18
64	-	8	=	□	"	□	x	8	=	64

(4) measurement and partition concepts

Division has been taught previously in terms of measurement (referred to also as containing) and partitioning (or sharing).

12 - 4 = □ can be used in either a measurement or a partition problem.

Examples:

1. A row of trees will contain four trees. How many rows can be made with twelve trees?
2. Twelve trees are to be placed in three equal rows. How many trees will be in each row?

In the first problem, the size of the group is fixed, but the number of groups is unknown. We want to know how many equal groups can be formed from one group. This is the measurement, or containing, interpretation. In the second problem, the number of resulting groups is fixed, but the size of each group is unknown. Many experiences should be provided with situations involving measurement and partition. The objective is to have pupils recognize that these different situations have a common solution.

- Have pupils decide whether the measurement or partition concept is used in the following situations:

  (a) 7 houses on a block. How many blocks will be needed for 28 houses?

  (b) 12 seats in a row at the movie theater. How many rows will there have to be to accommodate 60 seats?

  (c) 32 desks are put in groups of 4. How many groups will there be?

  (d) If 25 pennies are put in stacks of 5, how many stacks will there be?

b. <u>Develop division of 3-digit numbers by 1-digit numbers</u>.

(1) by estimation

Encourage pupils to estimate answers before actually computing. Rounding to the nearest tens or hundreds is an important first step in estimating.

Problem	Rounded	Estimate
$2\overline{)175}$	$2\overline{)180}$	90
$3\overline{)154}$	$3\overline{)150}$	50
$4\overline{)325}$	$4\overline{)320}$	80
$5\overline{)117}$	$5\overline{)100}$	20
$8\overline{)514}$	$8\overline{)480}$	60
$6\overline{)312}$	$6\overline{)300}$	50
$3\overline{)148}$	$3\overline{)150}$	50

Use the calculator to see if the estimate is reasonable.

(2) zeros in the quotient

- Stress with pupils that if the divisor does not go into the part of the dividend being divided then a zero is needed in the quotient.

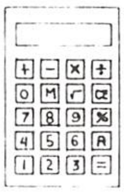
```
      2 0 8              4 0 5
  3 / 6 2 4          2 / 8 1 0
      6                  8
        2 4                1 0
        2 4                1 0
          .                  .

            2 1 0
        4 / 8 4 0
            8
            . 4
              4
              . 0
```

- Find the missing digits with zeros in the quotient.

$$6\overline{)\square\square\square} = 106$$

$$2\overline{)\boxed{7}\boxed{2}\boxed{6}} = 3\square\square$$

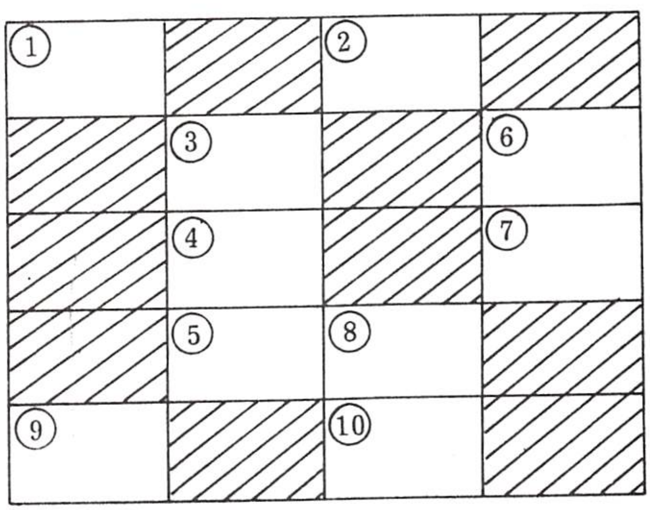
$$5\overline{)95\square} = \square 90$$



Use the calculator to have pupils check their calculations.

(3) writing remainders

- Fill in the puzzle with the remainders of these examples.



Answers	Answers
(1) 537 ÷ 2 (R.1)	(6) 410 ÷ 6 (R.2)
(2) 491 ÷ 8 (R.3)	(7) 627 ÷ 4 (R.3)
(3) 204 ÷ 7 (R.1)	(8) 452 ÷ 3 (R.2)
(4) 842 ÷ 5 (R.2)	(9) 257 ÷ 4 (R.1)
(5) 585 ÷ 7 (R.4)	(10) 612 ÷ 3 (R.0)

- Use counters to find remainders.

Provide a small group of students with 134 counters (graph paper, blocks, bundles of coffee stirrers) and have them complete the following chart.

Number of Stacks	Number in Each	Remainder
3	44	2
4	33	2
5	26	4
6	22	2
7	19	1
8	16	6
9	14	8

- Using money to find remainders.

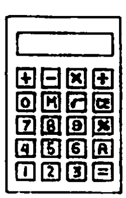
Here is a collection of money:

[$1] [$1] [$1] [$1] [10¢] [10¢] [10¢] [1¢] [1¢]

Divide it evenly between students. Ask them how much each one gets and whether there is any money left over. Divide the same money evenly among 4 students and ask how much each one gets and whether there is any money left over. Then divide the money among 5 students and ask the same questions.

- Have pupils complete the following table, finding remainders:

Dividend	273	324	750	368	202	564
Divisor	4	5	6	7	8	9
Quotient						
Remainder						



Check work with a calculator.

(4) checking

- Provide pupils with the opportunity to check their division examples by multiplying the quotient by the divisor.

  The work would look like the following example.

```
  1 9 4
3/5 8 2
  3
  2 8
  2 7
    1 2
    1 2
```

Check:

```
1 9 4  ———→ quotient
x   3  ———→ divisor
5 8 2  ———→ dividend
```

- Fill in the missing numerals in each of the following examples. Check each answer either with paper and pencil or with a calculator.

```
   1□4
6/ 7 4 4
   □
   1 4
   1 2
     2 4
     □
       □
```

```
   □1□
8/ 9 0 4
   8
   1 0
   □
     2 4
     2 4
       0
```

```
   □4 4
3/ 7 3 2
   6
   1 3
   □
     1 2
     □
       0
```

```
    2 3 3
4/ □ 3 □
   □
   1 3
   □
     1□
     1 2
       0
```

- Find the quotient and then check each example:

Dividend	738	364	576	270
Divisor	2	4	8	5
Quotient				
Check				

2.75 Mathematical Sentences

a. Develop the language and skills for solving mathematical sentences.

(1) Symbols

(a) Placeholders: The placeholder may hold a place for numerals, operation signs, or relation symbols. Geometric shapes such as squares, circles, and triangles (also known as frames) are used to provide a place in which to write a missing numeral or sign. Letter symbols may also be used as placeholders for numerals, but these are recommended at a higher level.

(b) Operation Signs: +, -, x, and ÷ .

(c) Relation symbols: These are used to compare numbers: = (equals), > (is greater than), < (is less than), and ≠ does not equal) are relation symbols which are the "verbs" of mathematical sentences.

(d) Grouping symbols: ( ) The parentheses are used to show the order of operations in those cases in which two or more operations occur in the same sentence. The expression within the parentheses is usually performed first. Using parentheses helps to clarify the meaning of the sentence.

(2) Open Sentences

An open sentence has one or more unfilled placeholders. It corresponds to an interrogative sentence. For example, $\square + 4 = 9$ asks the question, "Four added to what number equals 9?" In the open sentence at this level we are primarily interested in those replacements which yield true statements. When the placeholder is filled with a replacement which yields a true statement, we may say the solution has been found or that the equation has been solved. Such number sentences as $35 + \square = 72$ are called equations. The equation implies that numerals on either side of the = sign represent different names for the same number. We may read the equation either from left to right or from right to left; thus,

16 + 14 = 30 may be read as "16 plus 14 equals 30" or as "30 = 14 + 16."

(a) Varied use of frames

The placeholder in the open sentence may appear in different positions in the equation and may represent any of the mathematical symbols.

(b) Parentheses

Exercises in the use of parentheses are necessary to show that the order of operations can sometimes affect the solution. For this reason, the teacher should help pupils to discover that the operation within parentheses is usually done first.

5 - 3 + 2 = ☐

Think: "I see that two operations must be performed. I can read the equation in several ways. Five minus 3 (answer 2), plus 2, equals 4. Or, I can read the equation as 5 minus three plus 2 (5), equals 0. My answer depends on how I group the terms of the equation."

Develop the idea that mathematicians use parentheses to help clarify the meaning of such a sentence. When parentheses are used, we think of the sentence as a two-step sentence, and we perform the operation with the parentheses <u>first</u>.

24 - (12 - 6) = ☐

Think: "When I see parentheses, I know that my problem has more than one step. First I will find a simpler name for (12 - 6). 12 - 6 is 6. I can now do the second step. 24 - 6 is 18. ☐ = 18."

Provide practice in finding the missing number first for one operation, then for mixed operations.

37 - (6 + 7) = ☐

The same thought pattern is applied here even though two different operations are involved.

Emphasize the fact that the parentheses are also used in the equation to name a number in a more convenient way:

9 x 45 = ☐

9 x (40 + 5) = ☐

Forty-five has been written as 40 + 5. The value of 45 has not been changed but the parentheses indicate that for some mathematical reason, we have renames the number 45.

To strengthen the concept that the same number must be named on either side of the equation if the sentence is to be true, exercises of the following type should be performed.

Write the missing numeral:

67 + 86 = 29 + ☐

243 - ☐ = 203 + 15

254 x 4 = ☐ x 508

72 ÷ 8 = 18 ÷ ☐ .

Again, the pupil is required to do two-step thinking to solve the problem.

The thought patterns and the computation should proceed from the simple to the complex. When a new concept is being developed, the computational aspects should be kept simple. After the generalization has been reached, application may be at increasingly complex levels.

Teachers can use the open-sentence technique in unlimited ways. Children are highly motivated and challenged when drill is presented in a variety of settings and is not confined to the traditional algorithm form. The technique also provides an excellent means for translating verbal problems into mathematical language:

Numeral	Operation Sign	Relation Symbols
□ + 37 = 242	205 □ 37 = 242	205 + 37 □ 242
205 + □ = 242		
205 + 37 = □		

Provide similar practice.

The placeholder can represent a particular number. It can also represent numbers pupils have not yet been taught.

1. □ + 324 = 600	(Only 276 satisfies this.)
2. □ x 1 =	(Any number can replace this: the identity element for multiplication)
3. 2 - 6 = □	(Negative integer needed to solve this; children have not yet worked with integers.

Provide similar practice examples.

Two or more variables, or placeholders may be used in the same sentence, but a rule for replacement must first be established. The rule of like replacements holds for placeholders of the same shape.

If the same-shaped placeholder occurs more than once in the sentence, the same number must be used to fill all of the same-shaped placeholders. When the placeholders are different, the numbers may be different.

1. □ + □ + 44 = 144
2. □ + □ + △ = 110

In the first example, the placeholders must each be replaced by 50 to solve the problem. In the second example, many replacements will satisfy the condition.

[40] = [40] = △30 = 110 or [35] + [35] + △35 = 110 etc.

Although other "rules" can be established for the solution of equations, it is recommended that such rules not be taught at this time.

Open sentences with two different shapes may be filled with any pairs of numbers that will fit, and the numerals may be recorded in an organized manner.

△ x □ = 120

△	□
60	2
40	3
15	8

etc.

The pupils apply intuitive methods to solve the equations in initial work with open sentences. Through discovery, they can arrive at principles which can be applied to equations of the same patterns.

Have pupils work with types of equations which can readily be solved by intuitive methods:

7 + □ = 12

Think: "7 plus what number = 12? I know that 7 + 5 = 12. Therefore, the missing addend is 5. I can also find my answer by subtracting 7 from 12."

From repeated experiences with the simple equation, the teacher should lead pupils to generalizations which can be applied to more complex sentences:

37 + □ = 346

Think: "To find a missing addend, I subtract one addend from the sum. 346 minus 37 = 309. The missing addend is 309."

The teacher should not proceed too rapidly through the various stages of development and must provide practice to develop understanding of the symbols.

329 + 275 + □ = 613	Missing addend
15 + (76 - □) = 76	Subtraction as inverse
□ x △ = 72	Drilling number facts
(30 △ 6) x 20 = 100	Determining operation
5 x 90 = (5 x □) x 10	Associative property
□/1 = □	Generalizations about dividing by 1

The sentences should be an integral part of the teaching program, but should also be treated in a developmental way.

(3) Equalities and inequalities

When an equation has been solved and we have asserted that the symbols on either side of the equal sign are names for the same number, we say that the sentence is an equality.

When the relationship between the symbols on either side of the equation is not equal, this can be expressed by one of the relation symbols: $\neq$ (does not equal), $>$ (is greater than), $<$ (is less than). We say that sentences containing these symbols are inequalities.

If the $\neq$ symbol holds for a sentence, we know that the number represented on one side of the $\neq$ sign is not equal to what is expressed on the other side. The number on one side must therefore be greater than ($>$), or less than ($<$) the number on the other side.

Provide practice in interpreting relations symbols. Direct pupils to tell what each symbol means:

$4 \neq 5$	Think, "Four does not equal five."
$5 > 2$	Think, "Five is greater than 2."
$2 < 5$	Think, "2 is less than 5."

Reading from left to right, we remember the hint that the relations symbols $>$ and $<$ always point to the smaller of the two quantities compared.

Provide practice in symbolizing the inequality of two quantities.

Place a $>$ or $<$ sign in each circle:

3 + 2 ◯ 8	17 - 4 ◯ 8
17 ◯ 10 + 4	8 - 5 ◯ 11 - 4
24 + 12 ◯ 15 16	22 - 17 ◯ 27 - 10

To find the value of a missing number in initial work dealing with inequalities, pupils use intuitive methods.

> 5	Think: 5 equals 5, so I can use any number greater than 5 to make the sentence true.

$\square + 4 > 6$

Think: 2 + 4 equals 6. I am looking for a number which will make the left side of my sentence greater than 6. I know that if I use any number greater than 2 as my missing number, then the left side of the sentence will be greater than 6 and the sentence will be true.

Many experiences of this type will help pupils discover that they should first solve the inequality as though it were an equality. Assuming that all of the numbers with which they are familiar are whole numbers, they can then easily determine the missing number. They discover, too, that there are usually many numbers which will satisfy the inequality. The concept of inequality is very useful in developing the division algorithm; the thinking involved in exercises dealing with the concept will help pupils apply it to the division situation. Exercises using multiples of 10 and 100 will help to insure this:

Select the correct number and put it in the box:

10	x	$\square$	>	35	(4, 5,...)
100	x	$\square$	>	460	(5,6,...)
10	x	$\square$	<	62	(6, 5, 5,...0)
100	x	$\square$	<	725	(7, 6, 5,...0)

(4) True and false sentences

A mathematical sentence can be true, false, or neither.

An open sentence is neither true nor false since it contains a variable. When the variable is replaced by the name of a number, it becomes true or false. To illustrate:

$\square + 6 = 9$	Neither
$3 + 6 = 9$	True
$3 + 6 = 10$	False

We cannot determine whether the first equation is true or false because a solution has not been given. We know that the second equation is true because 3 + 6 is another name for 9. We know the third equation is false since 3 + 6 is not another name for 10. Equations can be true or false. Inequalities can be true or false. Begin the sequence of establishing this concept with statements such as the following:

Tell whether the statements are true or false.

Our flag is red, white, and blue.

Carrots are blue.

Shoes are worn on your feet.

Your stomach is in your mouth.

Tell whether the equations are true or false.

$15 + 16 = 31$

$17 - 9 = 10 - 2$

$3 \times 4 = 12 \div 2$

$45 + 13 = 35 + 21$

To establish the idea that inequalities may be true or false, begin with statements that are inequalities:

Tell whether the statements are true or false:

A cow does not wear shoes.

Potatoes are not food.

Philadelphia is not a city.

A pencil is not for sewing.

Tell whether the inequalities are true or false.

$7 > 5$

$4 + 16 \neq 20$

$60 + 40 < 70 + 90$

$23 + 45 \neq 60 + 74$

Exercises involving this concept should be kept simple, with emphasis on equations that are true or false. The concept of true and false inequalities will be developed to a greater depth at a higher level. The open sentence approach to true and false inequalities is also recommended for the higher level.

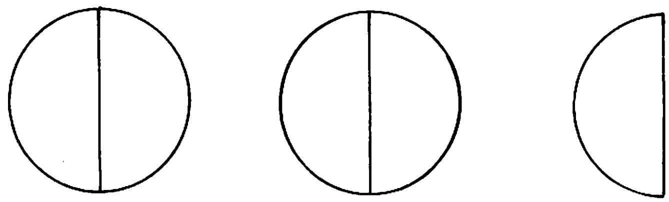
FRACTIONS

3.10 Common Fractions	3.20 Decimal Fractions
a. Explore physical models for introducing fractional numbers > 1. (1) congruent regions (2) numberline b. Develop notation for fractions > 1 (1) mixed numbers (2) improper fractions	a. Develop decimal and common fraction equivalents; denominators of 10 or 100. b. Review addition and subtraction of U.S. money. c. Extend rounding off to nearest hundredth.

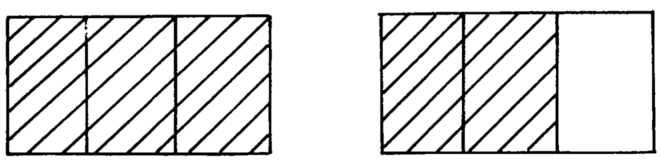
3.10 Common Fractions

a. Explore physical models for introducing fractional numbers > 1.

(1) congruent regions



Draw three circles on board and erase half of one circle. Ask the class: How many whole circles are there? (2) How much is remaining? ($\frac{1}{2}$) How many halves are in 2 circles? (4) How many halves are in $2\frac{1}{2}$? (5)

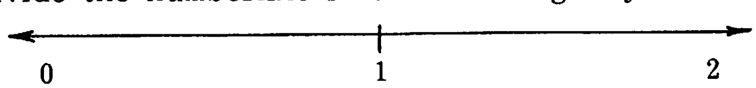


Have pupils describe the shaded areas. Ask: How many congruent regions make up a unit figure? (3) How many congruent regions have been shaded? (5) To describe this, the fraction 5/3 is used.

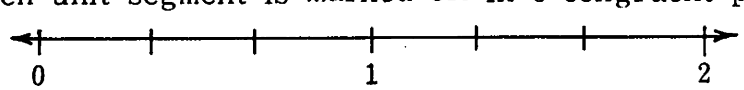
Have pupils use index cards to represent candy bars. Cut them into thirds so that pupils see that $2 = \frac{6}{3}$ and that $2\frac{2}{3} = \frac{8}{3}$.

(2) numberline

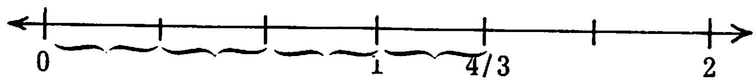
The numberline may also be used to consider fractions whose values are greater than 1. In this case the numberline will include more than 1 unit interval; the unit intervals will each be divided into congruent parts. To locate 4/3 on the numberline, you would divide the numberline in the following way:



Each unit segment is marked off in 3 congruent parts.



To locate 4/3, then, 4 of the congruent parts are counted; the point which has been located in this way is the point associated with 4/3.



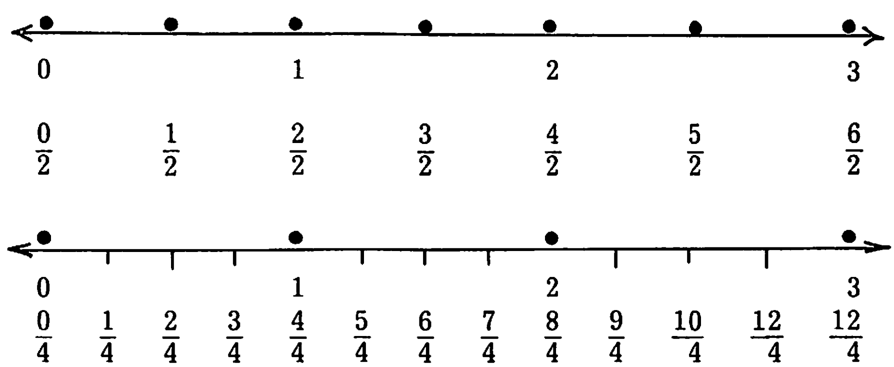
Similarly, points on the numberline can be associated with 12/5, 10/3, 8/6.

The teacher will want to provide many opportunities for these notions to be developed in an intuitive way.

b. Develop notation for fraction > 1.

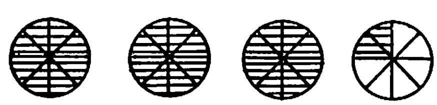
(1) mixed numbers

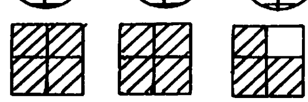
Prepare a worksheet with two numberlines, each labeled only with the whole number 0, 1, 2, 3. Divide the first numberline into intervals of $\frac{1}{2}$ unit, and the second into intervals of $\frac{1}{4}$ unit. Help pupils label the first line by halves from $\frac{0}{2}$ through $\frac{6}{2}$. Have them use the numberline to find the mixed number equivalent to $\frac{3}{2}$ ($1\frac{1}{2}$). Continue with other mixed forms that have denominators of 2 or 4.



Have pupils complete exercises such as the one below:

Write a mixed numeral.

1.  $3\frac{2}{8}$

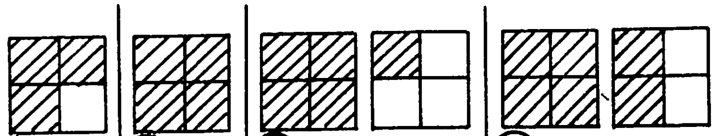
2.  $2\frac{3}{4}$

3. 5 and $\frac{7}{8}$ $\quad 5\frac{7}{8}$ 4. 10 and $\frac{4}{5}$ $\quad 10\frac{4}{5}$

5. $34 + \frac{2}{3}$ $\quad 34\frac{2}{3}$

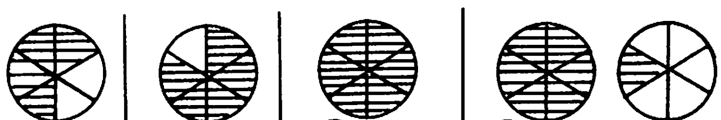
(2) improper fraction

Have pupils complete an exercise such as this: Study these examples. Write the fraction which is related to each picture:



(1) 3/4 | (2) 4/4 | (3) 5/4 | (4) 6/4

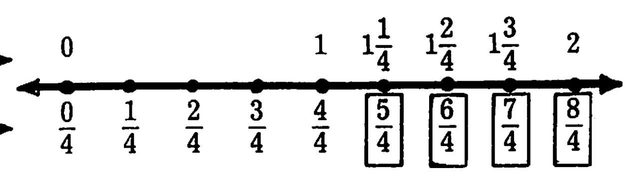
Write the fraction suggested by each exercise:



(1) 4/6 | (2) 5/6 | (3) 6/6 | (4) 7/16

MIXED NUMBERS ⟶

FRACTIONS: ⟶

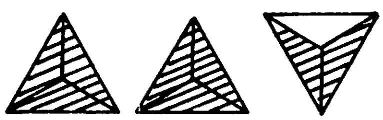


Display a numberline on chalkboard.

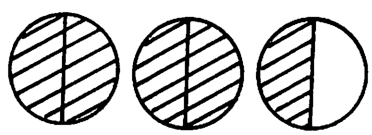
Have pupils fill in the notation for mixed numbers and improper fractions. Ask questions such as the following:

What is a fraction for $1\frac{1}{4}$?

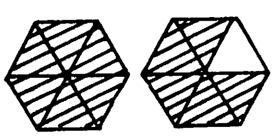
What is the mixed number for $\frac{7}{4}$?

1. 

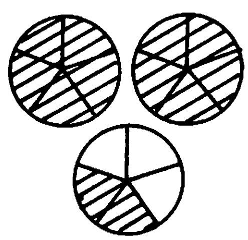
$\frac{8}{3} = 2\frac{2}{3}$

2. 

$\frac{5}{2} = 2\frac{1}{2}$

3. 

$\frac{11}{6} = 1\frac{5}{6}$

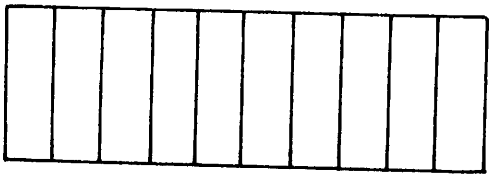
4. 

$\frac{12}{5} = 2\frac{2}{5}$

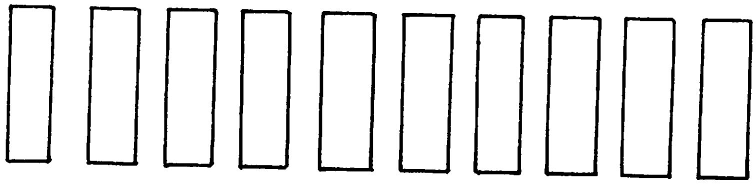
Have pupils write a number sentence to show the part shaded. Use an improper fraction and a mixed number, as shown in the examples.

3.20 Decimal Fractions

a. Develop decimal and common fractions equivalents; denominators 10 or 100.



Start with a piece of graph paper



Cut into 10 parts of the same size.

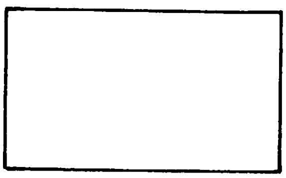
Each part is one-tenth of the whole. Ten-tenths make one whole. Pupils will use the paper strips to show three-tenths, seven-tenths and nine-tenths.

Use a numberline.

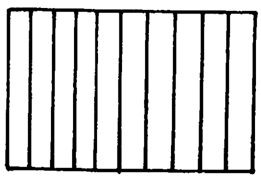
Fractions	$\frac{0}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$
Decimals	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

Have pupils read the fractions, then read the decimals. Explain that the name for each part can be written two ways ($\frac{3}{10}$ and 0.3).

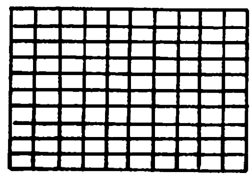
- The development for decimal and common-fraction equivalency with hundredths parallels the lesson for tenths.



Start with a piece of graph paper.



Cut or shade tenths.



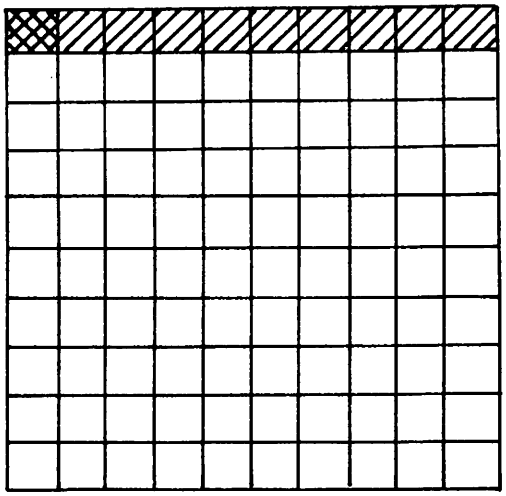
Cut or shade each tenth into 10 parts of the same size.

Ten hundredths make one-tenth.

Ten tenths make one whole.

Write 0.03 and state that this numeral is read as "three-hundredths." Have pupils show three hundredths using the paper square hundredths. Have them write three-hundredths as a fraction ($\frac{3}{100}$). Repeat this procedure for other decimals showing hundredths.

- Help pupils draw a hundreds board. Have them shade in one square. Then ask how this part can be named using the common fraction and the decimal fractions. Record the notation.



$\frac{1}{100} = 0.01$

Develop a chart of equivalents:

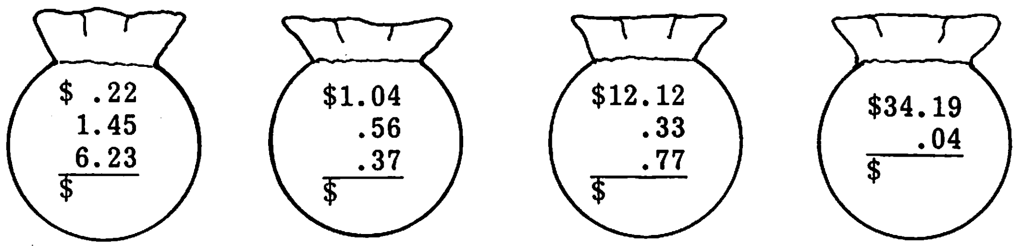
	Names for the Same Fractional Number:			
	Common Fraction		Decimal Fraction	
1.	$\frac{2}{10}$	$\frac{20}{100}$	0.2	0.20
2.	$\frac{4}{10}$	$\frac{40}{100}$	0.4	0.40
3.	$\frac{5}{10}$	$\frac{50}{100}$	0.5	0.50
4.	$\frac{7}{10}$	$\frac{70}{100}$	0.7	0.70
5.	$\frac{10}{10}$	$\frac{100}{100}$	1.0	1.00

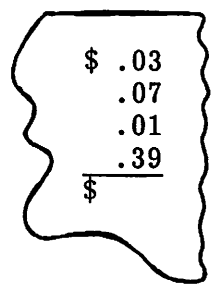
Use the calculator to show decimal and common-fraction equivalents, e.g., to show $\frac{3}{10} = 0.3$.

Key-in 3 [÷] 10 [=]. Display will read 0.3. Provide additional practice.

3.20 b. Review additional and subtraction of U.S. money.

- Draw the bags below on the board. Have pupils find which has the most money in it.





- Have pupils cut pictures of objects and prices (to $99.00) from newspaper ads and mount them on index cards. On each card, write an amount of money that is less than the price. This is the amount the pupil has saved. Pupils must subtract to determine how much more they need in order to buy an item.

c. Extend rounding off to nearest hundredth.

- Rounding with decimals is just like rounding whole numbers.

  Have the students study the chart below and read each line.

to round	to the nearest	look at the digit on the right.	If that digit is	
			<5 round to	= 5 or >5 round to
.53	tenth	.53	.5	
.37	tenth	.37		.4
.634	hundredth	.634	.63	
.488	hundredth	.488		.49
.155	hundredth	.155		.16
.013	hundredth	.013	.01	

3.95 Applications

- Provide word problems such as the following:

  Dave bought a book about boats for $9.00 and a book about camping for $6.79. How much did he spend altogether?

MEASUREMENT

4.20 Linear Measure

a. Review tables of linear measure.

b. Introduce the concept of mile.

c. Extend the use of the ruler to 1/8 inch.

4.30 Time

a. Review second, minute, hour, day, year.

b. Extend to century.

c. Develop telling time to the minute.

(1) A.M. and P.M.

4.60 Temperature

a. Review above and below zero readings on the thermometer.

(1) Celsius

(2) Farenheit

4.80 Weight, Liquid Measure

a. Review all measures and equivalents.

b. Introduce liter-kilogram relationship.

4.20 <u>Linear Measure</u>

a. Review tables of linear measure.

Review the customary units of linear measure and compose a table which can be posted for easy reference. Provide sufficient practice so that students learn these facts. The table should include these facts:

12 inches	=	1 foot
3 feet	=	1 yard
36 inches	=	1 yard

b. Introduce the concept of a mile.

Discuss the need for a unit to measure lengths and distances greater than a foot and greater than a yard. Ask when we might want to measure distances this great. Elicit that distances traveled by automobile, train, bus, trolley, or airplane would be difficult to express in feet or yards; very large numbers would be needed. Discuss the distances which are familiar to some of the students, such as from school to City Hall, or from Philadelphia to Atlantic City.

- If possible, take a class walk for one mile. It may take about 20 minutes.
- Add to the table of linear measure the following:

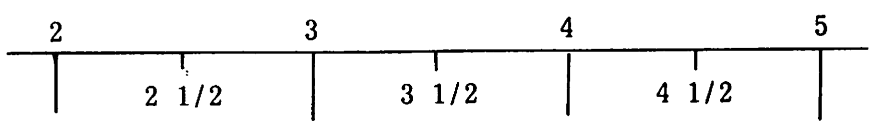
5280 feet	=	1 mile
1760 yards	=	1 mile

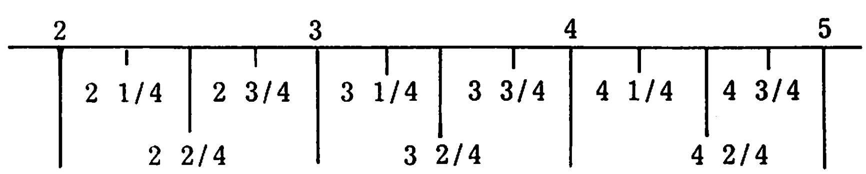
mi. is the abbreviation for mile.

Use appropriate test or duplicated materials as a follow-up.

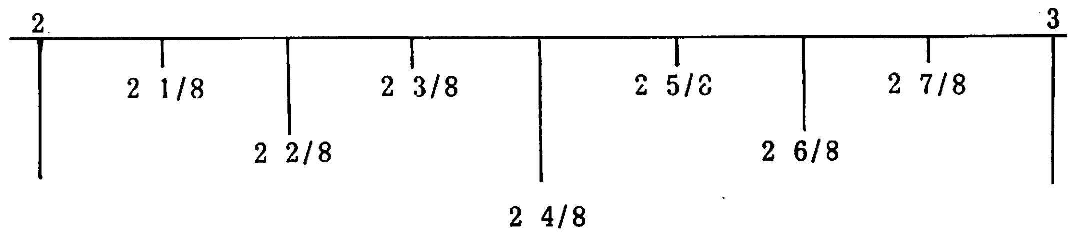
c. Extend the use of the ruler to 1/8 inch.

- Explain to pupils that we need a measure that is more precise when an item in customary measure is not a whole number of inches. To obtain this precision, we break our inch down into congruent parts. Review the breakdowns previously learned: 1/2 inch and 1/4 inch.

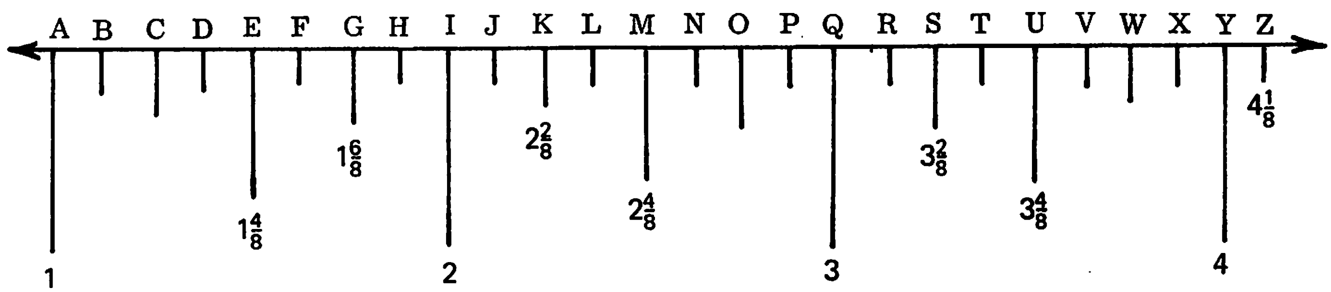




Show the placement of eighth-inch segments (not to scale).



- Show the relation of a ruler divided into eights to a fraction numberline.



Find a letter for each value listed. What word did you write?

$3\frac{2}{8}$ $1\frac{2}{8}$ $1\frac{7}{8}$ $2\frac{6}{8}$ $2\frac{6}{8}$ $2\frac{3}{8}$

___ ___ ___ ___ ___ ___

Find a value for each letter in this word:

M A T S

___ ___ ___ ___

- Provide opportunities to measure real objects in the classroom. Use appropriate textbook and duplicated material for reinforcement.

4.30 Time

a. Review second, minute, hour, day, year.

- Discuss with pupils the measures of time which would be appropriate for many of their activities. For instance: lessons are measured in minutes; the time spent in school is measured by days; birthdays are measured in years.

- Provide pupils with an opportunity to experience seconds and minutes by using a clock's sweep hand or a stop watch. Have pupils stand by their desks. Ask them to sit down ten seconds after you say "begin." Try again for 30 seconds, and again for one full minute. The activity usually takes less time than pupils estimate.

- Use appropriate activities and text materials to reinforce the following concepts:

60 seconds	=	1 minute
60 minutes	=	1 hour
24 hours	=	1 day
7 days	=	1 week
365 days	=	1 year
366 days	=	1 leap year
12 months	=	1 year
10 years	=	1 decade

  Pupils should be thoroughly familiar with these equivalences.

b. Extend to century.

- Add the following to the table in 4.30 a.

  100 years = 1 century

- Discuss centuries in terms of rounded numbers: 1400, 1500, 1600, 1700, 1800, 1900, 2000, etc. Pupils can verify the number of years in a century through simple subtraction:

$$\begin{array}{r} 1900 \\ -\ 1800 \\ \hline 100 \end{array}$$

- Ask questions such as:

  If you were born in 1971, when will you be 100 years old?

  How many years from 1776 to 1876?

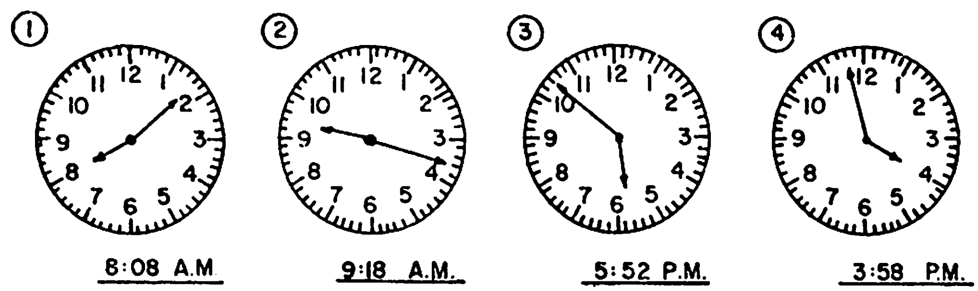
c. Develop telling time to the minute.

(1) A.M. and P.M.

- Pupils must become familiar with reading time to the minute on both standard clock faces and those with digital readouts.

- Review telling time in five-minute intervals. Review the fact that an hour has 60 minutes. In all problems dealing with time, pupils should be aware of whether it is A.M. or P.M. and should label all work appropriately. A.M. is used for times after 12:00 midnight and before 12:00 noon. P.M. is used for times after 12:00 noon and before 12:00 midnight.

- Examples such as the following should be presented on an overhead projector or with a class-size clock with moveable hands.



These may be read as:

1. Eight minutes after eight A.M.
2. Eighteen minutes after nine A.M.
3. Five fifty-two, or eight minutes of six P.M.
4. Three fifty-eight, or two minutes of four P.M.

For some pupils, it may be necessary to point out that in reading time written with the colon, (e.g., 3:15 P.M.), the numeral before the colon shows the hour and the numeral after the colon designates the number of minutes past the hour.

- Use every opportunity possible in daily class activities for pupils to read the classroom clocks. Making pupils responsible for class schedules will provide practice in reading clocks to the minute.

- Provide pupils with follow-up using appropriate text and duplicated materials.

4.60 Temperature

a. Review above- and below-zero readings on a thermometer.

  - Provide large thermometers for pupils to practice reading the temperature.

- Display a large cardboard thermometer indicating both Fahrenheit and Celsius scales. Change the temperature setting several times during the day. Have someone record under the thermometer the temperature it is showing, e.g., 34° or minus 20°C.

(1) Celsius

- Have pupils record as part of their homework the temperature given on the news. Keep a weekly log of the temperature in Celsius.

- Ask pupils to think about different places in the world that always have temperatures below zero degrees Celsius, other places where the temperature would be above zero always, and places where the temperature would fluctuate.

Examples: Temperature in Celsius Degrees:

Below 0°C	Above 0°C	Above and Below 0°C
Antartica	Panama	Philadelphia
North Pole	Puerto Rico	Chicago

- On an outside thermometer, have pupils read various temperatures telling whether it is below zero degrees or above zero degrees Celsius. At several times during the day, have a pupil read and tell the class the temperature. This can be recorded to find out at what time of the day the temperature was the warmest or coolest. It can also be used to see between what hours the temperature changed the most. This activity can be done several times during the year. There will be times when the temperature is below zero degrees and times when the temperature is above.

- Show pupils a chart with temperatures such as:

	40°C	very hot	above zero
	33°C	hot	above zero
	17°C	warm	above zero
minus	10°C	cold	below zero
minus	24°C	very cold	below zero

Have pupils read each temperature, and tell whether it was above or below zero and how they know.

Have pupils explain how the temperature reflects weather that is hot, warm, cold, etc.

- Ask pupils to suggest some activities that they participate in when the temperature is above or below zero. Make a chart showing these activities:

  Things to do when the temperature is

above 0°C	below 0°C
riding a bike	ice skating
swimming	making a snowman
hiking	riding a sled
going on a picnic	looking for icicles

- Have pupils find and record various temperatures:
  - temperature in the classroom ____________
  - temperature of cold water in the drinking fountain ______
  - temperature of hot water from a faucet ___________
  - their body temperature___________
  - outside temperature___________

- Have pupils compare temperature by drawing a ring around the one that is warmer or around the one that is colder:

18°C	or	20°C
16°C	or	6°C
minus 15°C	or	minus 20°C
0°C	or	10°C
8° below zero	or	0°C

(2) Fahrenheit

All activities shown above using the Celsius thermometer can be used with the Fahrenheit thermometer. Only minor adjustments may need to be done.

4.80 Weight, Liquid Measure

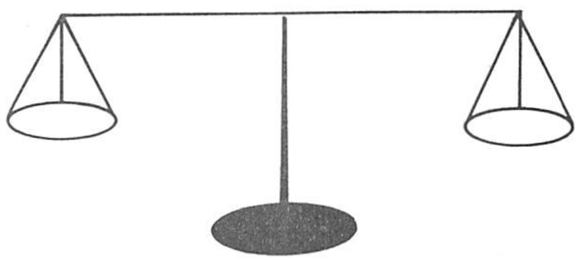
a. Review all measures and equivalents.

- Before completing activity below, remind pupils that 1 lb. = 16 oz., ½ lb. = 8 oz., and ¼ lb. = 4 oz.

  Have pupils bring from home labels showing weights. The labels can be put into categories for a display such as:

GREATER THAN ONE POUND	ONE POUND	BETWEEN ONE-HALF POUND AND ONE POUND	ONE-HALF POUND	LESS THAN ONE-HALF POUND

- Help to balance the scale by filling the blanks correctly.



18 ounces = 1 lb. ______ oz.
24 ounces = ______ lb. ______ oz.
32 ounces = ______ lb.
3/4 lb. = ______ oz.
3/8 lb. = ______ oz.
$1\frac{1}{2}$ lb. = ______ oz.

An actual balance scale can be used to check answers.

- Display a list of things that are usually measured in ounces and in pounds:

ounces	pounds
candy bar	chicken
piece of fruit	people
______	______
______	______
______	______
______	______

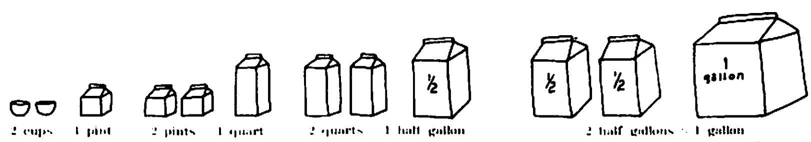
- Ask pupils to recall liquid measure learned previously. A list can be displayed in the room. Try to get as many equivalents as possible. Some of them might be:

2 cups	=	1 pint
2 pints	=	1 quart
1 cup	=	$\frac{1}{2}$ pint
$\frac{1}{2}$ quart	=	1 pint
1 quart	=	$\frac{1}{4}$ gallon

- Have pupils compare liquid measures. Select the greater or lesser amount.

7 pints	or	3 quarts
6 cups	or	4 pints
9 pints	or	1 gallon
1 gallon	or	10 cups
12 pints	or	2 gallons
$\frac{1}{2}$ gallon	or	3 quarts

- Have pupils complete a chart as shown below. Provide opportunity for pupils to actually measure different amounts.



Make each amount with as few containers as possible.

Quantity	Cups	Pints	Quarts	Half-Gallons	Gallons
1 cup					
2 cups					
3 cups					
4 cups					
5 cups					
6 cups					
7 cups					
8 cups					
9 cups					
10 cups					
11 cups					
12 cups					
13 cups					
14 cups					
15 cups					
16 cups					

- Complete tables such as the following:

Gal.	Pints
1	—
—	24
5	—
—	16

Qts.	Gal.
2	—
—	3
8	—
1	—

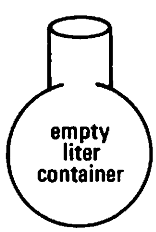
Pints	Qts.
1	—
—	4
—	2
3	—

b. Introduce liter-kilogram relationship.

- At this time, pupils know that a <u>liter</u> is the unit used to measure liquids, and that the <u>gram</u> is the unit used for measuring mass (weight).

- Explain that there is a definite relationship between units in the metric system. Show the relationship between the liter and the kilogram.

- Measure the mass (weight) of an empty liter container. Fill the container with water and then weigh it again. The mass will be one kilogram plus the mass (weight) of the empty container.



32 grams

1000 grams (1 kilogram)
plus 32 grams

Repeat this experiment using various liter containers.

- Have pupils take liters of various liquids (milk, juice, soda) and weigh them to see if the relationship holds between the liter and kilogram. Be sure that pupils take into consideration that the container has mass of its own.

- When pupils have the idea of the relationship between the liter and kilogram, have them predict the mass of:

  8 liters of water = __________ kg

6 liters of water	=	kg
2 liters of water	=	kg
5 liters of water	=	kg

- Have pupils estimate the number of liters of water weighing

  4 kilograms

  1 kilogram

  7 kilograms

  3 kilograms

ORGANIZING AND INTERPRETING DATA

5.10 Graphs	5.20 Tables and Charts
a. Interpret and construct simple bar graphs. b. Review interpretation and construction of simple line graphs.	a. Interpret and construct tables and charts.

5.10 Graphs

a. Interpret and construct simple bar graphs.

In earlier levels, pupils had experiences organizing data into graphs where the vertical axis increased by one unit. Familiar data such as daily temperatures, time, birth months, and attendance were used as items for graphing. At this level, the vertical axis of the graph can show an increase of two units.

Graphs constructed by the teacher or pupils should have:

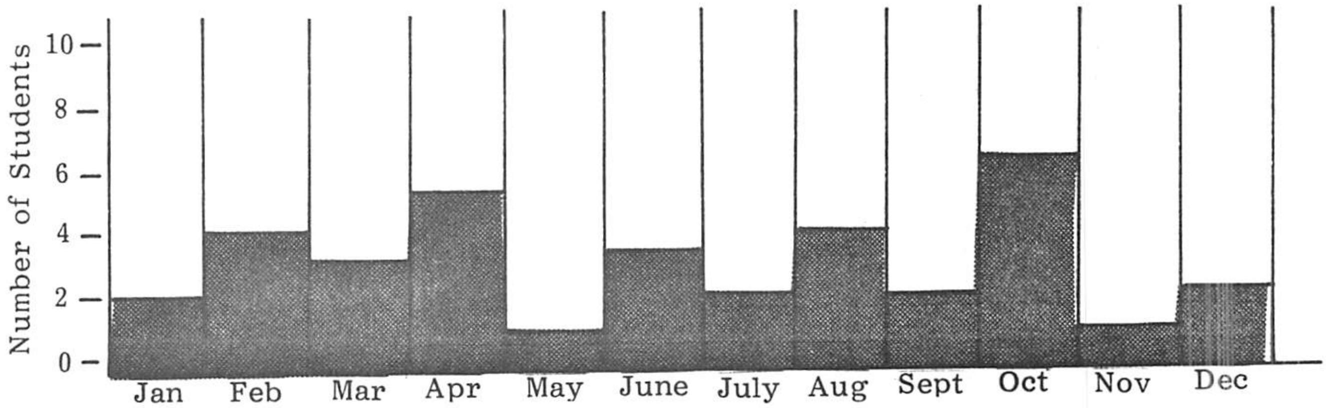
A title indicating the information on the graph.

Labels for the vertical and horizontal axes.

A vertical axis numberline beginning at zero.

All graphs should be constructed on grid paper or grids on the chalkboard. Pupils must understand that the interval on the vertical axis does not always show a numeral for each bar. In such instances, they must estimate the number.

The teacher should ask appropriate questions about each graph.

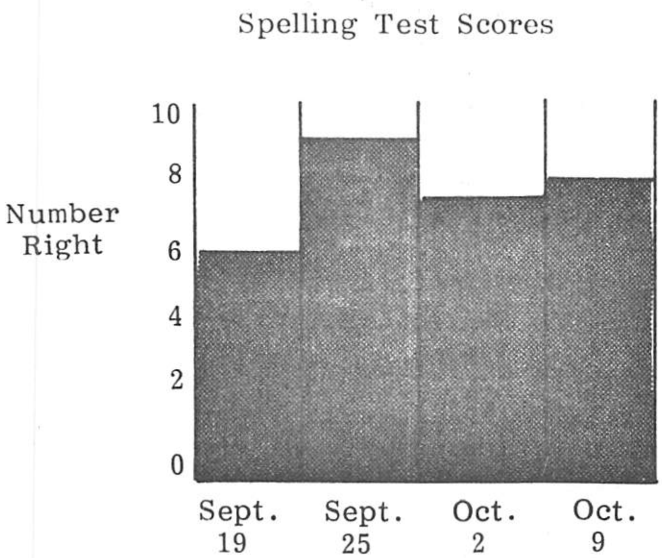


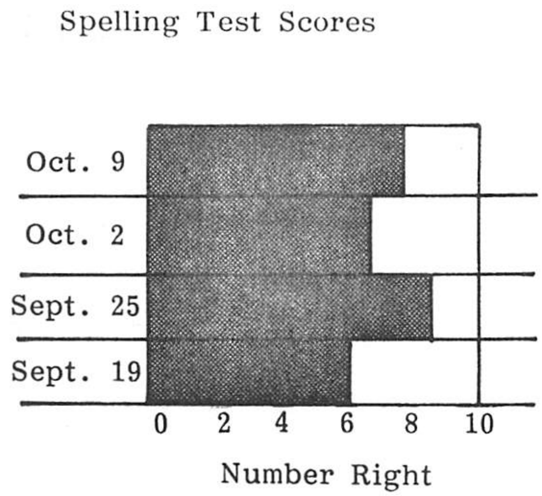
Birth Months in Our Class

Questions:

1. How many students in this class have birthdays in October, May, Jan.? (6,3,2)
2. In which months do the same number of students have birthdays? (Feb., Aug.) (Jan., July, Sept., Dec.) (Mar., June) (May, Nov.)
3. What is the total number of birthdays in April and October?
4. How many more students have birthdays in October than in September? (4)

- The vertical bar graph is more widely used than the horizontal form. However, the horizontal bar graph can be introduced using data identical to that in a vertical bar graph.





- Construct a vertical bar graph representing the following data: (Use an interval of 2.)

  Mary's Arithmetic Scores:

  Monday, 18; Tuesday, 15; Wednesday, 12; Thursday, 17.

  Answer the following questions:

  1. On which day did the lowest score occur? (Wednesday)
  2. On which date did the highest score occur? (Monday)
  3. How many more points were scored on Monday than on Wednesday? (6)

  Construct a horizontal bar graph representing the same data.

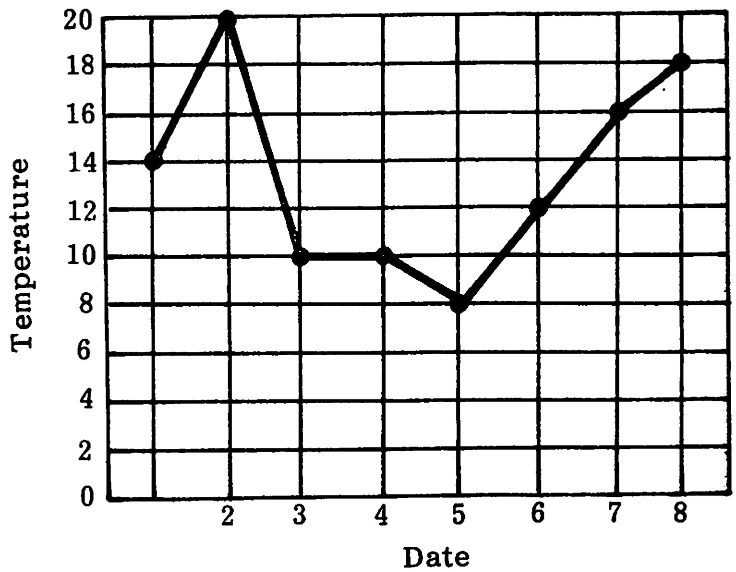
- Construct vertical and horizontal bar graphs for the favorite subjects of students in the class:

  mathematics, reading, art, music, gym, science

- Construct vertical and horizontal graphs for the number of blocks students in the class walk to school.

b. Review interpretation and construction of simple line graphs. For some data, it is necessary to use a line graph instead of a bar graph. A line graph is used to show changes in data. Scores or necessary elements are recorded as points. The points are then connected with line segments. A line graph is illustrated below.

Low-Temperature Recordings: Feb. 1 to Feb. 8



The teacher should develop appropriate questions for use of the graph:

1. On which two dates were the temperatures the same? (3rd, 4th)
2. On which date was the temperature the lowest? (5th)
3. On which date was the temperature the highest? (2nd)
4. What was the change in temperature from the lowest to the highest degrees? (12 degrees)

- Have pupils make a line graph showing the temperature for five consecutive school days at a given time of day.
- Have pupils make a line graph showing the temperature at hourly intervals on a given day.

5.20 Tables and Charts

a. Interpret and construct tables and charts.

An example of data which may be presented in chart form is a class record of performance in physical-fitness tests.

	Push-ups		Sit-ups		Pull-ups	
	OCT.	APR.	OCT.	APR.	OCT.	APR.
Albert	3	6	12	16	0	1
Charles	7	12	20	20	3	5
Frank	2	3	10	14	1	2
James	5	6	14	20	2	4
Robert	1	1	10	11	0	0
PHYSICAL FITNESS SCORES - BOYS - ROW 4						

Sample questions for interpreting the chart:

1. Who did the most push-ups?

2. Who showed the most improvement in pull-ups?

3. How much improvements did James make in sit-ups?

- Newspapers and almanacs may be used as sources of data for charts and tables:

  Total month rainfall for each month in more than one location. Daily high and low temperatures for a week or month in a given location. Use the Celsius scale.

GEOMETRY

6.70 <u>Developing Geometric Concepts</u> a. Review concepts (1) point (2) line (3) segment (4) plane (5) space (6) simple closed curve b. Introduce concept and notation for ray. c. Introduce the concept of an angle. d. Introduce line relationships. (1) intersecting (2) parallel (3) perpendicular e. Introduce the concept of an angle. (1) types (a) right (b) acute (c) obtuse	

6.70 <u>Developing Geometric Concepts</u>

a. Review concepts.

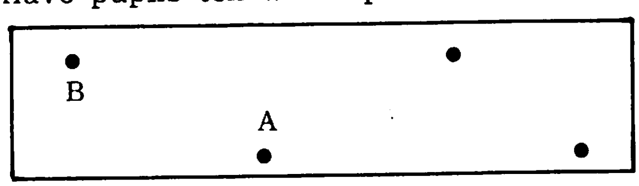
(1) point

The geometric point represented by the dot is an exact location which cannot be seen because it has no size or shape.

Hold a photograph of a celebrity for pupils to identify. After the identification has been made, help pupils recognize that a picture of a person is not the person; it is only a representation of a person. In a like manner, a dot is a representation of a geometric point.

In geometry we usually label points with capital letters.

Have pupils tell which points are named in the following picture.

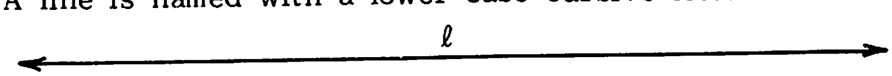


Use letters to name the other two points.

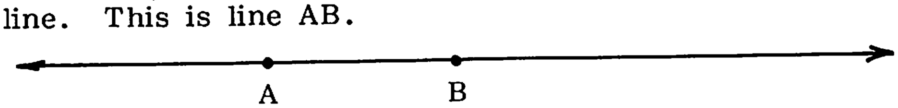
(2) line

A line in geometry is undefined and may be described as a set of points which extend endlessly in both directions. Unlike a line segment, a line has no endpoints.

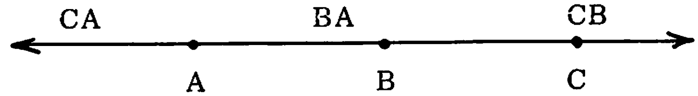
We represent a line by drawing a line segment and attaching arrows to both ends to indicate that it continues endlessly in both directions. A line is named with a lower-case cursive letter.



A line may also be named by any line segment contained within the line. This is line AB.



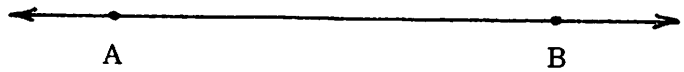
The line below has many names: $\overleftrightarrow{AC}$ $\overleftrightarrow{AB}$ $\overleftrightarrow{BC}$ or $\overleftrightarrow{CA}$ $\overleftrightarrow{BA}$ $\overleftrightarrow{CB}$



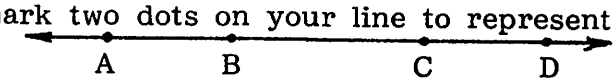
A line may be thought of as an extension of line segments.

Have students complete the following exercise:

- Use a straightedge to make a representation of a line on your paper.



Now mark two dots on your line to represent points A and B.



1. Name the line above. (The line can be named using any two letters.)

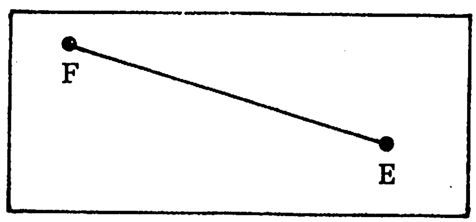
   $\overleftrightarrow{AB}$ $\overleftrightarrow{AC}$ $\overleftrightarrow{AD}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{BD}$ $\overleftrightarrow{CD}$

2. How many lines can be drawn through any two points? (one)

(3) segment

A special path or curve which is the most direct path between two points is called a line segment.

- To develop the idea of line segments, have pupils place two dots on a piece of paper. Label the two dots.



Connect the two dots using a straightedge and pencil.

The capital letters of the endpoints of the line segment name the line segment. For example, the above line segment would be named "line segment FE." The line segment ends at points F and E. We refer to F and E as endpoints. A short way to wrote ";ome seg,emt FE" is FE. EF is another way to name line sigment FE.

- Provide opportunities for pupils to recognize representations line segments.

Examples:

An edge of writing paper

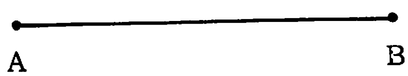
An edge of a ruler

The crease of a folded paper

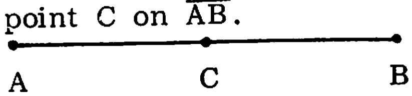
Pupils should understand that one line segment can contain other line segments.

Have pupils follow the following directions:

Draw a line segment on paper. Label the endpoints A and B.



Place a point C on $\overline{AB}$.

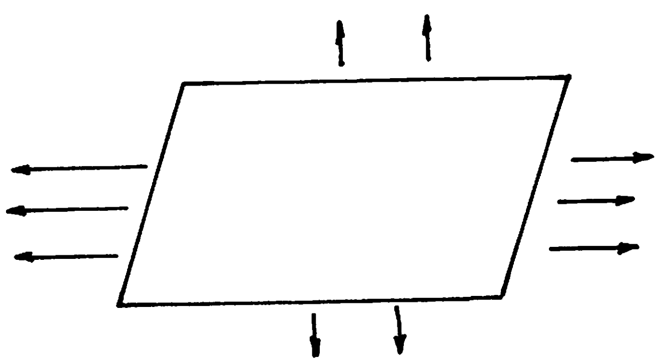


Name the three line segments.

$\overline{AC}$ $\overline{CB}$ $\overline{AB}$

(4) plane

A plane is an endless extension of a particular set of points or locations in space. Because a plane has no limits, any representation can only show part of a plane. In geometry, the idea of the plane is best shown by any flat surface such as a sheet of paper, a table top, the floor, or a desk top. If we use a flat surface of a table to represent a plane, the table surface represents only part of that plane. Pupils must use their imaginations to understand that a plane extends endlessly in many directions.



The picture above represents a plane; this plane extends infinitely.

The use of many physical representations will help the children gain an intuitive knowledge of planes. The teacher might find these suggested activities helpful:

- Begin by giving each child a 3 x 5 card. Explain that this flat surface represents a plane. Have the children replace the card with a sheet of writing paper. This is a larger representation of a plane. Direct their attention to the surface of the desk.

Encourage children to select flat surfaces in the room that represent a plane.

Examples:

Floor

Ceiling

Chalkboard

Surface of a table.

(5) space

Once the concept of a point is understood, the teacher should introduce the concept of geometrical space. Space is the set of all points. Another way of saying this would be "space is the set of all exact locations everywhere." All objects from the very smallest to the very largest cover or occupy space.

- Have pupils make a short list of things that occupy space.

  At home

  In school

  In Philadelphia

  In the United States

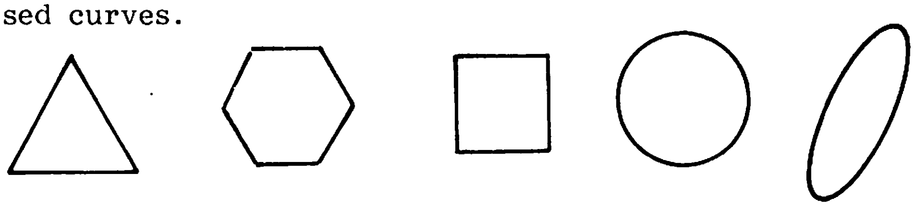
  In the world

  In the universe

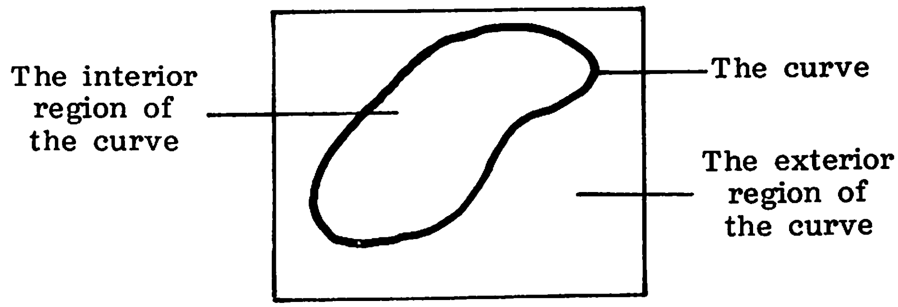
(6) simple closed curve

Review that in geometry all paths whether "straight" or "curved" are called curves.

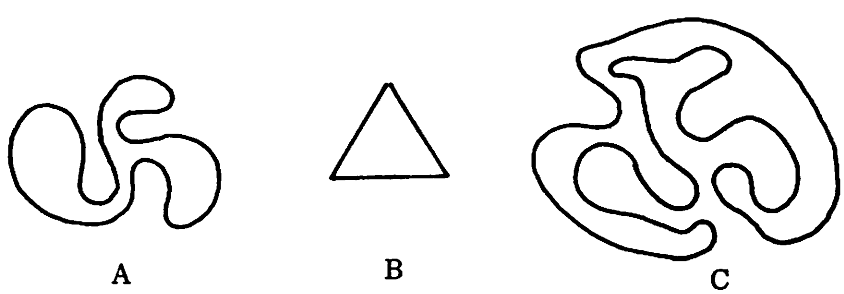
A simple closed curve is a set of points lying in the same plane which has the same beginning and ending point and does not intersect or cross itself. Any polygon or circle is a representation of a simple closed curve. The figures below are examples of simple closed curves.



A simple closed curve that lies in a plane divides the plane into three sets of points: the points on the curve, the points outside the curve, and the points inside the curve. The points on the curve are called the <u>curve</u>. The points outside the curve are the <u>exterior region of the curve</u>. The points inside the curve are called the <u>interior region of the curve</u>.



- Draw a representation of a triangle on the chalkboard. Tell the pupils that the chalkboard represents a plane, and that the triangle represents a simple closed curve. Call on individual pupils to place a point on the exterior region and on the curve of the triangle. Label the points.
- Review polygons such as square, rectangle, triangle, hexagon as simple closed curves.
- Have pupils draw simple closed curves without lifting the pencil from the paper. Display the resulting designs on a bulletin board.

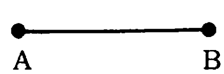


b. Introduce concept and notation for ray.

The concept of a ray should be introduced before the introduction of the concept of an angle. A method for introducing the concept of a ray follows:

Give these directions to students

(1) Draw line segment AB.



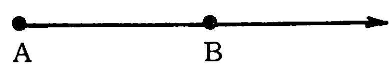
(2) Draw a point C such that B is between A and C.

(3) Draw a point D such that B is between A and D.

(4) Continue in this fashion.

(5) Describe what you have. (What you want to elicit from pupils is that the figure will consist of a segment, and all the points to one side of one of the endpoints such that the other point is between them and the other endpoint.

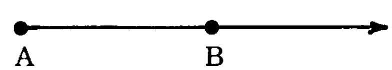
To show that the points continue infinitely, we draw:



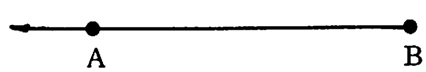
This is called a ray. Ray AB consists of the segment AB and all the points such that B is between them and A.

The beginning point of the ray is named first. An arrow over the letters is used to indicate a ray.

$\overrightarrow{AB}$ is read as "ray AB"



Ray BA ($\overline{BA}$) begins at B and includes all points such that A is between them and B.



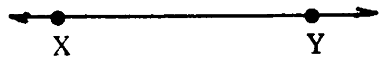
The teacher may use the beam of a flashlight to help pupils gain insight about a ray. A beam of light begins at the flashlight and continues on and on into space. Another physical representation of a ray would be the rays of light emanating from the sun. The flashlight and the sun represent the endpoint from which a ray begins.

- Activity:

1. Draw $\overline{XY}$

2. Draw $\overrightarrow{XY}$

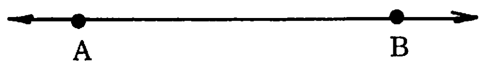
3. Draw $\overleftrightarrow{YX}$



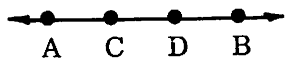
4. What do you have? ($\overleftrightarrow{XY}$) - "line XY")

- Activity:

1. Draw line AB ($\overleftrightarrow{AB}$)



2. Draw points C and D on $\overleftrightarrow{AB}$



3. Name as many rays as you can. Answers $\overrightarrow{AC}$ ($\overrightarrow{AD}$ and $\overrightarrow{AB}$ are names for the same set of points), $\overrightarrow{CD}$, $\overrightarrow{DB}$, $\overrightarrow{BD}$, $\overrightarrow{DC}$, $\overrightarrow{CA}$.

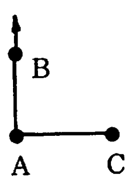
c. Introduce the concept of an angle.

(1) Types

(a) Right

Define an angle as a set of points formed by the union of two rays which have a common endpoint.

- Using a sheet of rectangular paper, demonstrate that each corner of the paper has a special L-shaped look.

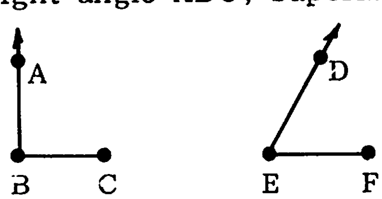


Angle BAC (BAC) represented above is formed by the union of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Point A is the common endpoint for both rays and is called the vertex of the angle.

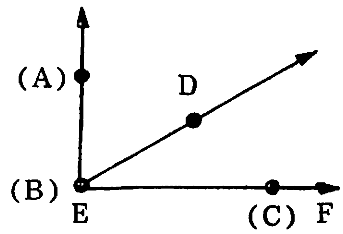
Lead pupils to an intuitive discovery of a right angle by having them observe the angles formed by the intersection of the edges of a wall and a floor, by the intersection of a table top and a leg, or by the intersection of a ceiling and a wall.

(b) Acute

Using right angle ABC, superimpose another angle DEF on it.



The superimposition of ∡ DEF on ∡ ABC is shown below.



Notice that $\overrightarrow{ED}$ lies in the interior of ∡ ABC. We say that the measure of ∡ DEF is less than the measure of ∡ ABC. Correctly written, using the symbols of inequality, the mathematical sentence would read:

m (∡ DEF) < m (∡ ABC).

For convenience, as long as we understand that the measures of the angles are being considered, we may write the sentence in this form:

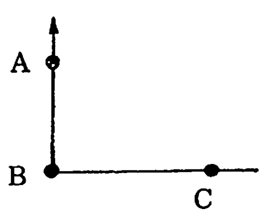
∡ DEF < ABC ∡

Care should be taken to avoid confusion in using the sign of inequalities < > and the symbol for angles ∡ .

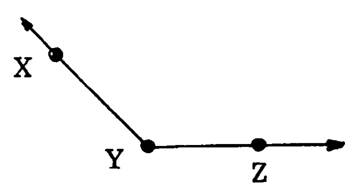
In out comparison of ∡ DEF and right angle ABC, ∡ DEF was found to be smaller than ∡ ABC. Any angle which is less than a right angle is called an <u>acute angle</u>.

(c) Obtuse

The procedure described above may be used to develop the concept of an obtuse angle. One ray of the obtuse angle will fall in the exterior of a right angle. A right angle and an obtuse angle are shown below:

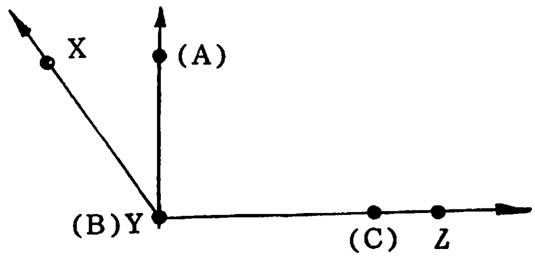


Right Angle



Obtuse Angle

If we superimpose ∡ XYZ on right angle ABC, the superimposition would appear as below;



Ray XY lies in the exterior of $\angle$ ABC; therefore, $\angle$ XYZ is larger than $\angle$ ABC. We may write this as a mathematical sentence:

$\angle$ XYZ $\qquad\qquad$ $\angle$ ABC

Pupils should be led to discover that any angle which is larger than a right angle is called an <u>obtuse angle</u>.

1. Draw several kinds of angles on the chalkboard. Have pupils identify them as right angles, acute angles, or obtuse angles.

2. Number the angles on the chalkboard and have the students write mathematical sentences about pairs of angles such as:

   $\angle$ ABC $<$ $\angle$ DEF
   $\angle$ XYZ $<$ $\angle$ ABC

3. Have students search for representations of angles in the classroom and identify them as being right angles, acute angles, or obtuse angles.

4. Direct the students to draw and label various plane figures such as triangles, rectangles, squares, etc.

   Name the vertices of each figure.

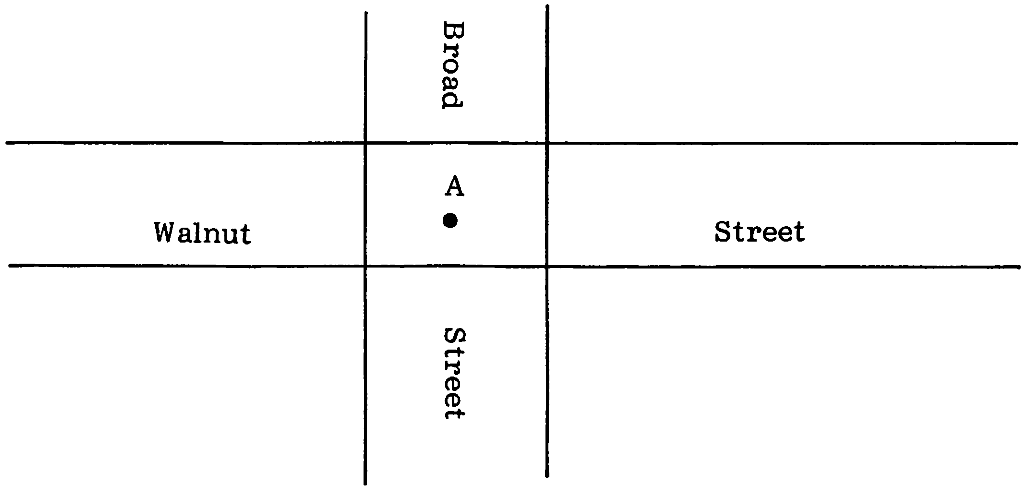
   Name the angles of each figure.

   Classify angles (acute, obtuse, right).

d. Introduce line relationships.

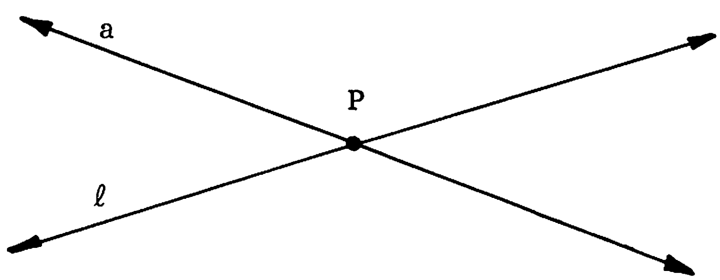
(1) intersecting

- A teacher might begin the intuitive development of the intersection of two lines by having the pupils think about two local streets that intersect or cross. The development might be similar to the one which follows:



1. What two streets are represented on the chalkboard?
2. What name do we give to the place where they meet? (Crossing is acceptable. Try for intersection).
3. If you were to stand at the spot represented by point A, on which street would you be? (both streets)
4. Direct a pupil to shade in with colored chalk the area common to both streets.
5. Lead pupils to generalize in their own words that these two streets provide an area common to both streets and intersect only once.

At this point, recall with the pupils that a geometric line is a set of points in space which extends endlessly in two directions. What can be observed if two different points in the same plane intersect?



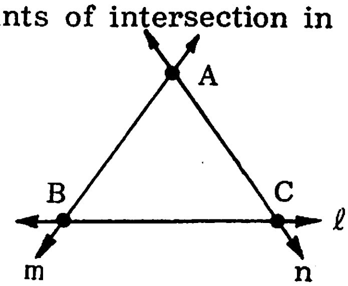
Line a and line ℓ intersect at point P. Point P is an element of the set of points represented by line a. It is also an element of line ℓ. Since point P is common to both sets of points, the intersection of a and ℓ is point P. Ask students to determine how many points of intersection there are for two intersecting lines in a plane.

- Have class do the following activity:

  Using a ruler, draw two intersecting lines. Label each line and the point of intersection.

  List three examples of models of intersecting lines, such as ceiling tiles.

  Name the points of intersection in this diagram:



(2) parallel

In the development of intersecting lines, it was noted that two lines intersect in one and only one point. However, it is possible that two lines, lying in the same plane, will never intersect no matter how far they are extended. Such lines are named parallel lines.

- Pupils may be led to an intuitive understanding of parallel lines by observing the edges at the top and bottom of a book page and the two edges of a ruler or a yardstick. Ask leading questions such as:

  1. What does the top edge of your page represent? (line or segment or line)
  2. What does the bottom edge represent? (line segment or line)
  3. Do the two edges intersect? (no)
  4. Do they have any point or points in common? (no)
  5. Would the edges intersect if they were extended infinitely? (no)

  Lead the pupils to generalize in their own words that two lines in a plane which are parallel do not intersect regardless of their extension in space.

  Have pupils list examples of parallel lines found in their environment, such as railroad tracks and ladder rungs.

- Have pupils do the following activity:

  1. Using a ruler, draw line segments of equal length along the upper and lower edges of the ruler. Label the two parallel line segments.

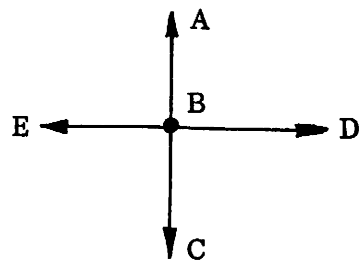
2. Draw a rectangle. Label the points of intersection. List the pairs of parallel line segments.

3. Draw a plane figure which has two sides parallel and two sides that are not.

(3) perpendicular

If two intersecting lines form right angles, then the lines are perpendicular lines.

- Copy the diagram below on the chalkboard:



$\angle ABD$, $\angle ABE$, $\angle EBC$, $\angle DBC$ are right angles.

Illustrate the following to the students:

$\overleftrightarrow{AC}$ and $\overleftrightarrow{ED}$ are intersecting lines that form four right angles. The point of intersection is B. Therefore, $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{ED}$.

- Have students list examples of perpendicular lines in the school environment, such as adjacent edges of rectangular objects.

PROBLEM SOLVING
IX

7.20 Strategies	7.30 Analysis of Word Problems
a. Extend the use of diagrams to solve problems.	a. Continue selection of pertinent data from problems. b. Identify problems with sufficient/insufficient data. c. Review concept of labeling where necessary.

7.20 <u>Strategies</u>

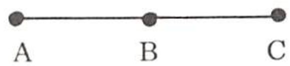
a. Extend the use of diagrams to solve problems.

Continue to practice solving problems by using diagrams. The following types of problems are suggested for use at this level:

Type I

How many segments can you name? 1

$\overline{AB}$

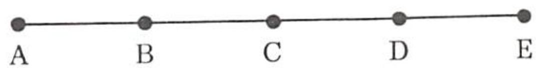


How many segments? 3

$\overline{AB}$, $\overline{BC}$, $\overline{AC}$

How many segments? 6

$\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{BC}$, $\overline{BD}$, $\overline{CD}$

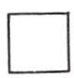


How many segments? 10

$\overline{AB}$,$\overline{AC}$,$\overline{AD}$,$\overline{AE}$,$\overline{BC}$,$\overline{BD}$,$\overline{BE}$,$\overline{CD}$,$\overline{CE}$,$\overline{DE}$

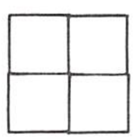
Type II

What is the total number of squares in a 1 x 1 array?



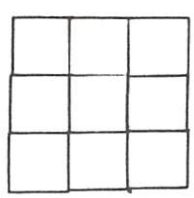
1

2 x 2 array?



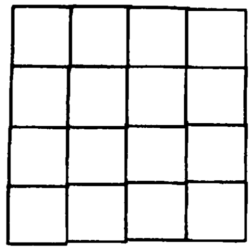
1 - 2 x 2
4 - 1 x 1
5

3 x 3 array?



1 - 3 x 3
4 - 2 x 2
9 - 3 x 3
14

4 x 4 array?



1 - 4 x 4

4 - 3 x 3

9 - 2 x 2

16 - 1 x 1

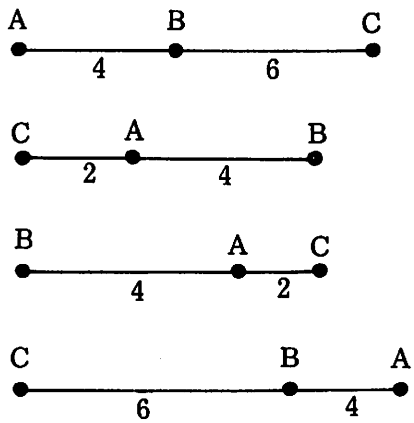
30

Type III

Three towns are all on a straight line. Town A is 4 miles from Town B. Town B is 6 miles from Town C. How far is Town A from Town C?

Answer: They could be 10 miles apart or they could be 2 miles apart.

Possible diagrams:



Answer: 10 miles

Answer: 2 miles

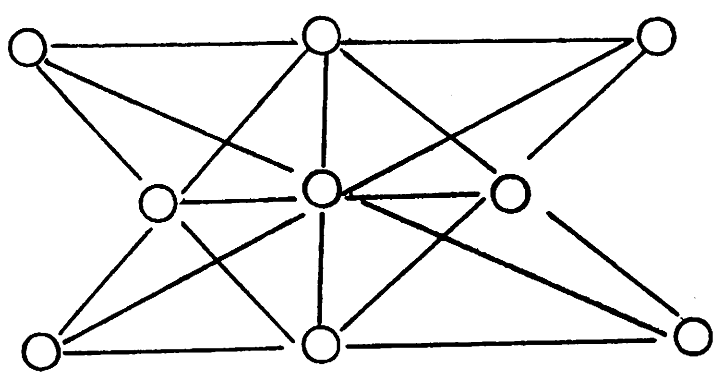
Answer: 2 miles

Answer: 10 miles

Type IV

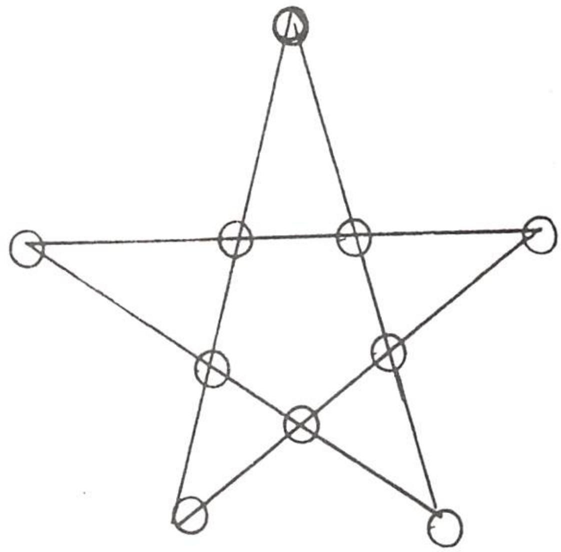
Arrange 9 circles in 10 rows of 3 each (for experts!)

Answer:



Arrange 10 circles in 5 rows of 4 each.

Answer:



7.30 Analysis of Word Problems

a. Continue selection of pertinent data from problems.

Selecting the necessary information needed for solution is basic to problem solving. Continue to stress selecting facts from a problem by using the following techniques: (Use original problems or those from textbooks.)

1. Have pupils fill in missing blanks:

Mary scored an 87 on a spelling test. Jane scored ____ on her test. What was the difference in their scores?	TEST SCORES Mary - 87 Jane - 76

2. Have students determine whether statements based on a problem are true or false.

> Dick had $1.25. Mary has 3 quarters, and Sue has $.55.
> How much money did they have between them?
> Mary had more than Sue. True or false?
> The girls had less than the boys. True or false?
> Their total was less than $3.00. True or false?

3. Ask questions about the information:

> Pencils cost $.15 each at the school store. Marge wants to buy 3 of them. She has a half-dollar. Does she have enough money?
>
> Cost of 1 pencil? ______________
>
> Cost of 3 pencils? ______________
>
> Money Marge has? ______________

7.30 b. Identify problems with sufficient/insufficient data.

Most textbook problems encountered by students contain enough information for solutions. In real-life situations, there sometimes is not enough information. Pupils need practice in analyzing problems to determine if there is sufficient data for solution.

> WHAT IS NEEDED?
>
> You have $1.32 and you want to buy 3 books. Can you?
>
> Check (✓) one box:
>
> ☐ size of each book
>
> ☐ price of each book
>
> If I buy a book for $1.59, how much will I have left?
>
> Check (✓) one box:
>
> ☐ amount of money to start with
>
> ☐ price of two books

SUFFICIENT OR INSUFFICIENT INFORMATION?

Provide problems in which students must determine if there is enough information for solution.

> Joe has $7.50 and wants to spend $1.69. How much does he have left?
>
> Check (✓) one box:
>
> ☐ sufficient data
>
> ☐ insufficient data
>
> Flight 709 left Los Angeles bound for New York. After 2 hours the plane had traveled 1200 miles. How many more miles does the plane have to travel?
>
> Check (✓) one box:
>
> ☐ sufficient data
>
> ☐ insufficient data

The next step is to have pupils decide what information is needed. Have them read the problems and decide whether sufficient data is given. If not, discuss possibilities for adding data.

> Several items in the sporting goods store were on sale. Tom had $15.00 to spend for a basketball and some tennis balls. How much did he have left after buying these items?
>
> Answer: INSUFFICIENT DATA
>
> Possibilities - price of basketball
> - price of tennis balls
> - number of tennis balls purchased

> Ralph drove 50 mph on a trip to his cabin. After 3 hours of driving, how much farther did he have to go?
>
> Answer: INSUFFICIENT DATA
>
> Needed - distance to cabin.

7.30 c. Review concept of labeling where necessary.

Review and practice the ability to determine if an answer needs a label or not. Choose problems such as the following:

> Calvin and Tom decided to combine their money to buy a record. Calvin had $3.57 and Tom had $4.75. How much did they have together? Focus on the question "How much did they have together". Since this question refers to money, labeling the answer with $ is necessary.

Other problems can be chosen from a text or devised by the teacher to show that labels are sometimes necessary and sometimes not.

> Jose had 35 marbles, but lost 12 of them. How many <u>marbles</u> did he have then?
>
> NO LABEL - Label is given in question.

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